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POLE-ZERO MODELING OF TRANSIENT WAVEFORMS: A COMPARISON OF METHODS WITH APPLICATION TO ACOUSTIC SIGNALS

by

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Pole-Zero Modeling of Transient Waveforms: A Comparison of Methods with Application to Acoustic Signals

by

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Submitted in partial fulfillment of the
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ABSTRACT

The modeling of damped signals as the impulse response of a pole-zero system is considered for a broad range of pole-zero modeling algorithms. The goal is to obtain the best possible fit between the model impulse response and the modeled signal. Prony's method, the least squares modified Yule-Walker equations (LSMYWE), iterative prefiltering, and the Akakie maximum likelihood estimator are compared on known test sequences for a variety of model degrading situations (e.g., additive noise) to develop an understanding of which methods are most suitable for modeling real world signals. A correlation domain version of interative prefiltering is also introduced. The most robust algorithms are determined to be LSMYWE using singular value decomposition and iterative prefiltering (including the correlation domain version). Modeling several laboratory generated short duration acoustic signals confirmed the robustness of LSMYWE and iterative prefiltering. It is shown that correlation domain iterative prefiltering outperforms standard iterative prefiltering when large model orders are required for accurate modeling. Shank's method was determined to be the most effective method of determining the zeros of a pole-zero model when a time domain match is required.

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I. INTRODUCTION

A. RATIONAL MODELING OF TIME SERIES DATA

The need to find a compact parametric representation for time series data arises in many fields of study. In electrical engineering, finding such representations, or models, appears under such topics as optimum control, optimum filtering, system identification, model order reduction, waveform encoding, and spectrum estimation. Other fields refer to such modeling as time series analysis or forecasting. In digital systems, the model parameters take the form of difference equation coefficients or, alternatively, transfer function coefficients. The most general form of a difference equation is one that uses both feed-forward and feedback coefficients. The corresponding transfer function is a ratio of two polynomials in the complex variable z and is refered to as a rational, or pole-zero, model.

Historically, the more general pole-zero model has been used in relatively few applications compared to all-pole (feedback only) and all-zero (feed forward only) models. In some cases, an all-pole or all-zero model is the most appropriate. More often, however, the all-pole or all-zero model is chosen because the optimal model estimation procedures are better understood and easier to implement than those for pole-zero modeling. This is particularly true for the all-pole case which, in many situations, can be determined by solving a set of linear equations. Two recent developments have led to increased activity in applying pole-zero models: (1) technological advances in digital hardware have dramatically reduced computation costs and, (2) a greater variety of efficient techniques for estimating pole-zero model parameters is now available.

Literally hundreds of pole-zero modeling estimation algorithms and applications have been published. The majority of these are built around probablistic or stochastic modeling techniques. This is because stochastic modeling is usually the most appropriate to forecasting [Ref. 1, 2, 3] and spectrum estimation [Ref. 4, 5, 6] where very little is known about the system input which produced the time series being modeled. A deterministic methodology has also been used in which the input and output time series are available and the linear system which 'best' (usually in a least squares sense) produces this cause and effect is determined. This approach is usually found under the topics system identification [Ref. 7, 8, 9, 10] and waveform encoding [Ref. 11]. Another large body of literature which exists in parallel to stochastic and deterministic modeling is that of reduced-order modeling [Ref. 12] which largely deals with the system control applications of pole-zero modeling.

Although stochastic and deterministic pole-zero modeling have the same goal and use the same mathematical techniques, the distinction is significant in the way data is treated and in the performance criterion postulated. Specifically, stationarity requirements and assumptions about the probability density functions of random processes in stochastic modeling can be limiting when dealing with real world systems. In contrast, deterministic techniques make no specific assumptions about the time series to be modeled except, of course, that the form of the model chosen is appropriate.

One particular deterministic modeling problem that has received little detailed attention is that of finding a pole-zero model when the time series being modeled is a transient, 'impulse response'-like waveform. Examples of situations where such models may be useful include in modal or shock analysis of mechanical systems [Ref. 13, 14], antenna response to electromagnetic pulses [Ref. 15], and wavelet estimation in siesmic studies [Ref. 16]. Pole-zero models make sense for transient waveforms because such a model is *structural* for many transient signals. That is, impulse

response like transients are usually the result of a system of oscillators which have been excited for a very short time period relative to the natural frequencies of the oscillators. Pole pairs in pole-zero models correspondingly represent the resonant frequencies of a digital system. System zeros allow the initial conditions or phasing of the resonant frequencies to be modeled. This combination of poles and zeros allows signals to be modeled based on time domain matching. When an effective time domain match is achieved many concerns about assumptions during modeling become moot: an effective time domain match has, by definition, effectively characterized the signal in question. Previous work has concentrated on finding only transient model poles [Ref. 17, 18, 19, 20] or concentrated on one particular technique of pole-zero impulse response matching [Ref. 21].

B. THESIS OUTLINE

This thesis provides a performance comparison of several pole-zero modeling procedures applied to the problem of model estimation from impulse response data. The procedures compared are chosen to form a cross section of optimality and computational complexity of available techniques. In Chapter II, the modeling procedures selected for study are described in detail. The remaining three chapters are concerned with comparing the performance of these modeling techniques and are organized as follows:

1. To study the specific modeling properties of each method, test impulse response sequences are modeled in Chapter Three. Test sequences are constructed to simulate the degradations likely to be present in 'real world' transient signals (e.g. noise, unknown model order).

2. Using the results obtained in Chapter III, laboratory generated acoustic transient data is modeled in Chapter IV. This data is considered to determine the

performance of various techniques when modeling the highly complex data characteristic of real world sources about which very little is usually known.

3. Chapter V summarizes the main conclusions drawn from the results in Chapters III and IV. Recommendations for further study are also presented.

II. POLE-ZERO MODELING

A. OVERVIEW—THE VARIETY OF TECHNIQUES

Choosing a pole-zero modeling technique from among the many available techniques can be difficult. The complexity, applicability, and demonstrated effectiveness of the different methods are not always well documented. Additional consideration of the many refinements that often evolve as a technique is applied to different problems can lead to a perplexing array of tools with which to attack the modeling problem. For the particular case of modeling by impulse response matching, little work has appeared which sorts out the strengths and weaknesses of available modeling techniques or, in fact, indicates which methods may or may not be applicable.

The basic scheme for fitting a pole-zero system impulse response to a given data sequence, $x(n)$, is illustrated in Figure 2.1a. This is sometimes referred to as the *direct* model. As we shall see, the formulation of this problem leads to a set of nonlinear equations which require the use of iterative techniques to solve. To overcome the complexities inherent in solving nonlinear equations, the impulse response matching problem can be reformulated as shown in Figure 2.1b. This may be referred to as the *indirect* method. Note that these two formulations are not equivalent: the error of the direct method of Figure 2.1a is $e_d(n) = x(n) - h(n)$ while the error for the indirect method of Figure 2.1b is $e_i(n) = b(n) - a(n) * x(n)$, where $h(n)$ is the impulse response of the pole-zero system being found, $b(n)$ is the corresponding sequence of numerator coefficients and $a(n)$ is the corresponding sequence of denominator coefficients. The solution of the indirect problem is considered suboptimal in the sense that, except

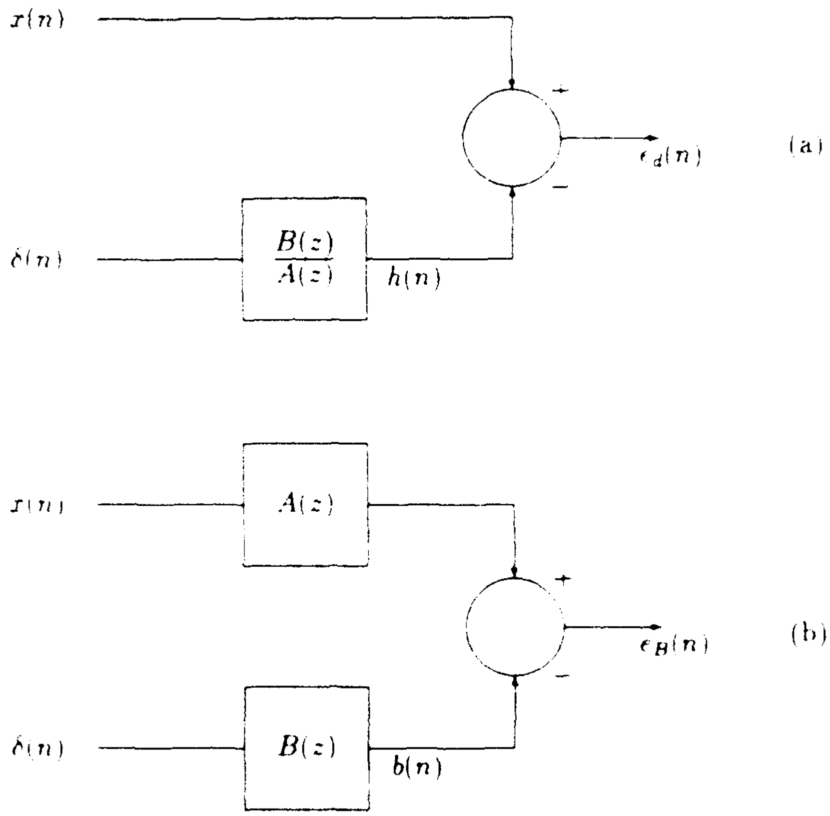


Figure 2.1: Two pole-zero modeling problem formulations: (a) direct formulation and (b) indirect formulation.

when the modeling error goes to zero, the effect of minimizing $e_i(n)$ is *not* the same as minimizing $e_d(n)$ in the direct method.

One way to organize pole-zero modeling techniques is shown in Figure 2.2. In deterministic waveform matching, the ideal equations relating the proposed model to the available input and output data sequences are formed. The solution which 'best' satisfies these equations is chosen. In this context, 'best' usually means the solution which minimizes the sum of squares of the equation error. These are essentially the Prony type methods [Ref. 17, 22] (when the input is an impulse response) and least squares system identification [Ref. 7, 8, 23] methods (for general input sequences). A number of iterative techniques for waveform matching have also been proposed. Waveform fitting error [Ref. 24, 25] and inverse filtering error [Ref. 26, 27] are the criteria most used.

Linear stochastic pole-zero modeling techniques rely primarily on estimates of second order statistics (auto- and cross-correlations) to estimate model parameters. Spectral estimation has been a driving force for these methods which are based on solving some form of the modified Yule-Walker equations (see Chapter III). Other methods which utilize reflection coefficients [Ref. 28] and higher order statistics [Ref. 29, 30] have also appeared.

The *maximum likelihood* technique seeks parameter estimates for which the observed data is the most probable in the sense that its conditional probability density function (likelihood function) is maximized. This technique is considered to be statistically optimum but is quite difficult to use because its implementation generally requires the minimization of a highly nonlinear function [Ref. 1, 31, 32, 33]. Four widely applied methods from each of the categories of Figure 2.2 will be used in this thesis. A number of improvements which have subsequently been suggested for these methods will also be considered. Most recent work in pole-zero modeling and

spectrum estimation has occured at the internal boundaries of Figure 2.2, i.e. equivalent linear techniques are sought that perform as well as modeling formulations that require solving nonlinear equations [Ref. 34, 35, 28, 36, 37]. The application of these newer techniques to transient modeling will not be considered here since we expect that the performance of the methods chosen will in most cases bracket the performance of these newer techniques with the possible sacrifice of computational efficiency. The four pole-zero modeling techniques chosen will be described in the remainder of this chapter. The transient modeling performance of these methods that will then be compared in Chapters III and IV.

	Linear	Iterative
Deterministic	Equation Error Methods	Waveform Matching Inverse Filtering
Stochastic	Correlation Equation Error Methods	Maximum Likelihood Methods

Figure 2.2: One way of organizing the various pole-zero modeling techniques.

B. THESIS MODELING TECHNIQUES

1. Prony's Method

One of the best known indirect techniques for matching a waveform to the impulse response of a linear time-invariant system is Prony's method [Ref. 22].

This method is in fact a special case of least squares system identification in which the system input sequence is taken to be a unit impulse and the numerator and denominator coefficients are determined separately.

The pole-zero modeling problem is formulated as follows. The time domain difference equation for a general feedback, feed-forward system can be written

$$\sum_{j=0}^{P} a_j x(n-j) = \sum_{i=0}^{Q} b_i u(n-i) \tag{2.1}$$

where $u(n)$ is the input sequence and $x(n)$ is the output sequence. When the input sequence is taken to be a unit impulse (unit sample function) and the output is taken to be the corresponding impulse response, (2.1) can be expressed in the form of a matrix equation,

$$\begin{bmatrix} x(0) & 0 & \cdots & 0 \\ x(1) & x(0) & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ x(N-1) & x(N-2) & \cdots & x(N-P) \end{bmatrix} \begin{bmatrix} 1 \\ a_1 \\ \vdots \\ a_P \end{bmatrix} = \begin{bmatrix} b_0 \\ \vdots \\ b_Q \\ 0 \\ \vdots \\ 0 \end{bmatrix} \tag{2.2}$$

where N is the number of data points used and, without loss of generalization, a_0 is set equal to one.

Equation (2.2) can be solved by partitioning,

$$\begin{bmatrix} \mathbf{X}_B \\ \mathbf{X}_A \end{bmatrix} \mathbf{a} = \begin{bmatrix} \mathbf{b} \\ \mathbf{0} \end{bmatrix} \tag{2.3}$$

where $\mathbf{a} = [1\ a_1\ \cdots\ a_P]^T$, $\mathbf{b} = [b_0\ b_1\ \cdots\ b_Q]^T$ and $\mathbf{X}_A$ and $\mathbf{X}_B$ are the corresponding lower and upper partitions of the data matrix in (2.2). The upper partition consists of the first $Q+1$ rows of the data matrix and the lower partition is composed of the remaining rows.

The solution can then be obtained by first solving the lower partition,

$$\mathbf{X}_A \mathbf{a} = \mathbf{0}, \tag{2.4}$$

for a and then finding b from the upper partition,

$$\mathbf{X}_B\mathbf{a} = \mathbf{b}. \tag{2.5}$$

If $N = P + Q + 1$ then (2.2) may have a unique solution and the model impulse response will exactly match $x(n)$ for $0 \leq n \leq P + Q + 1$. This is referred to as the Padè approximation [Ref. 22].

In most circumstances, however, the length of the available data sequence far exceeds $P + Q + 1$. It is then desirable to use all available data in setting up (2.2). This leads to an overdetermined set of linear equations for the lower partition. No exact solution to (2.4) usually exists in this case. The relationship in (2.4) becomes

$$\mathbf{X}_A\mathbf{a} = \mathbf{e}. \tag{2.6}$$

where $\mathbf{e}$ is the equation error that will be present. The solution of (2.4) and (2.6) requires the partitioning of $\mathbf{X}_A$ as follows.

$$\mathbf{X}_A = \left[\ \mathbf{x}_A \ \vdots \ \mathbf{X}'_A\ \right] \tag{2.7}$$

where $\mathbf{x}_A$ is the first column of $\mathbf{X}_A$. If the remaining matrix $\mathbf{X}'_A$ is square and of full rank, then the solution to (2.4) is given by

$$\mathbf{a}' = (\mathbf{X}'_A)^{-1}\mathbf{x}_A \tag{2.8}$$

where

$$\mathbf{a} = \begin{bmatrix} 1 \\ \mathbf{a}' \end{bmatrix}. \tag{2.9}$$

Otherwise the *least squares error* solution of (2.6), which minimizes the squared error $\mathbf{e}^T\mathbf{e}$, is given by

$$\mathbf{a}' = \mathbf{X}'^{+}_A\mathbf{x}_A \tag{2.10}$$

where $\mathbf{X}_A'^{+}$ is the Moore-Penrose pseudoinverse of $\mathbf{X}_A'$. When $\mathbf{X}_A'$ is of full (column) rank P, the psuedoinverse is given by

$$\mathbf{a}' = (\mathbf{X}_A'^T \mathbf{X}_A')^{-1} \mathbf{X}_A' \mathbf{x}_A. \tag{2.11}$$

(See [Ref. 4, pp. 28-33] for an example of this derivation). Otherwise, the pseudoinverse is defined by

$$\mathbf{a}' = \sum_{i=1}^{W} \frac{\mathbf{u}_i^T \mathbf{x}_A \mathbf{v}_i}{\sigma_i} \tag{2.12}$$

where σ_i, $\mathbf{u}_i$, and $\mathbf{v}_i$ are defined by the singular value decomposition (SVD) representation of $\mathbf{X}_A'$,

$$\mathbf{X}_A' = \sum_{i=0}^{W} \sigma_i \mathbf{u}_i \mathbf{v}_i^T. \tag{2.13}$$

The parameters σ_i are the singular values of $\mathbf{X}_A'$, $\mathbf{u}_i$ and $\mathbf{v}_i$ are the corresponding left and right singular vectors, and W is the rank of $\mathbf{X}_A'$. See [Ref. 38, Ch. 12] for a more detailed explanation of singular value decomposition. Once $\mathbf{a}$ is known, the upper partition of (2.2) can be solved by simply carrying out the matrix multiplication described by the left hand side of (2.5).

2. Modified Yule-Walker Equation Methods

A well known class of pole-zero modeling techniques is based of solving some form of the Modified Yule-Walker Equations (MYWE). These equations can be developed by multiplying (2.1) by $x(n-k)$ and taking the expectation of both sides. This yields,

$$\sum_{k=0}^{P} a_k r_{xx}(n-k) = \sum_{i=0}^{Q} b_i r_{ux}(n-i) \tag{2.14}$$

where $r_{xx}(l)$ is the autocorrelation sequence of the system output and $r_{ux}(l)$ is the crosscorrelation of the system input and output. If the original input to (2.1) is assumed to be a unit variance white noise sequence then the cross correlation, $r_{ux}(l)$, is given by

$$
\begin{aligned}
E[u(n)x(n-l)] &= E[u(n)\sum_k h(k)u(n-l-k)] \\
&= \sum_k h(k)\delta(l+k) \\
&= h(-l).
\end{aligned}
\tag{2.15}
$$

Equation (2.14) can be then be written as

$$
\sum_{k=0}^{P} a_k r_{xx}(n-k) = \sum_{i=0}^{Q} b_i h(i-n), \tag{2.16}
$$

or in matrix form,

$$
\left[\begin{array}{cccc}
r_{xx}(0) & r_{xx}(-1) & \cdots & r_{xx}(-P) \\
r_{xx}(1) & r_{xx}(0) & \cdots & r_{xx}(-P+1) \\
\vdots & & & \vdots \\
r_{xx}(Q) & r_{xx}(Q-1) & \cdots & r_{xx}(Q-P) \\
\hdashline
r_{xx}(Q+1) & r_{xx}(Q) & \cdots & r_{xx}(Q+1-P) \\
\vdots & & & \vdots \\
r_{xx}(Q+P) & r_{xx}(Q+P-1) & \cdots & r_{xx}(Q+1) \\
r_{xx}(Q+P+1) & r_{xx}(Q+P) & \cdots & r_{xx}(Q)
\end{array}\right]
\left[\begin{array}{c} 1 \\ a_1 \\ \vdots \\ a_P \end{array}\right]
=
\left[\begin{array}{c}
\sum_{i=0}^{Q} b_i h(i) \\
\sum_{i=1}^{Q} b_i h(i-1) \\
\vdots \\
b_Q h(0) \\
\hdashline
0 \\
\vdots \\
0
\end{array}\right]
\tag{2.17}
$$

As in Prony's method, the solution for the a_k and b_i coefficients can be determined separately. Taking a lower partition of the last P equations in (2.17) results in the matrix equation,

$$
\mathbf{R}_A \mathbf{a} = \mathbf{0}, \tag{2.18}
$$

where the theoretical values of the elements in $\mathbf{R}_A$ are replaced by estimated values.

If $P+1$ autocorrelation lags are used in constructing $\mathbf{R}_A$, then (2.18) can be solved directly for $\mathbf{a}$. However, if additional reliable lag information is available, we

will again desire to extend $\mathbf{R}_A$ by letting the index n in (2.16) run beyond $Q + P + 1$ resulting in additional equations. This leads to a an overdetermined set of equations.

$$\mathbf{R}_A \mathbf{a} = \mathbf{e} \tag{2.19}$$

which will not in general be satisfied with zero error. As before, application of the psuedoinverse results in a least squares solution of (2.19).

To understand how the MYWE methods can be used to match a time series to the impulse response of a pole-zero system observe that (2.16) describes the relationship depicted in Figure 2.3. This operation can be equivalently expressed as

$$r_{xx}(n) = h(n) * h(-n). \tag{2.20}$$

If the signal to be modeled is assumed to be a pole-zero system impulse response, then for the purpose of implementing (2.17), the signal being modeled can be substituted for $h(n)$ in (2.20).

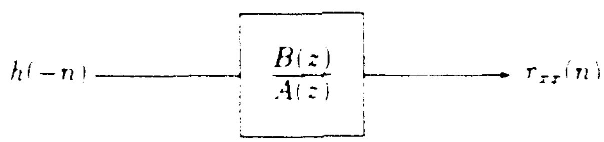


Figure 2.3: The system relationship described by (2.16).

The equations of the upper partition of (2.17) (the first Q equations) are not linear in the b coefficients and are generally not solved directly from (2.17). Once the denominator coefficients have been determined from (2.18) or (2.19), any of a number of techniques are available for finding the desired transfer function numerator coefficients. One method already discussed is to set up and solve the upper partition of Prony's method in (2.5). Three other techniques are spectral factorization, Durbin's method, and least squares identification, each of which are described below.

a. Spectral Factorization

Equation (2.16) describes the time domain relationship

$$r_{xx}(l) * a(l) = b(l) * h(-l) \tag{2.21}$$

where $a(l)$ and $b(l)$ represent the sequences of denominator and numerator transfer function coefficients, respectively. Taking the z-transform of (2.21) yields

$$S_{xx}(z)A(z) = B(z)H(z^{-1}). \tag{2.22}$$

Making the substitution

$$H(z^{-1}) = \frac{B(z^{-1})}{A(z^{-1})} \tag{2.23}$$

in (2.22) and rearranging, results in

$$A(z^{-1})S_{xx}(z)A(z) = B(z)B(z^{-1}). \tag{2.24}$$

To utilize (2.24) to find the polynomial coefficients of $B(z)$, which are the elements of the sequence $b(l)$, we must perform spectral factorization of the sequence resulting from the convolution of the three sequences $a(l)$, $a(-l)$, and $r_{xx}(l)$. A detailed explanation of this technique can be found in [Ref. 39] or [Ref. 40].

b. Durbin's Method

Durbin's method [Ref. 41] makes use of the property by which a process containing zeros can be represented by an all-pole system if enough poles are used. The first step is to filter the sequence to be modeled through the inverse of the previously determined all-pole filter coefficients as shown in Figure 2.4. The resulting residual sequence, $s(n)$, will nominally be an all-zero sequence. A large order all-pole model, $A_{large}(z)$, can then be fitted to $s(n)$ to obtain the relationship illustrated in Figure 2.5. If the model order for $A_{large}(z)$ is sufficiently large, all of the 'information' in $s(n)$ will be contained in the coefficients of $A_{large}(z)$. If an all

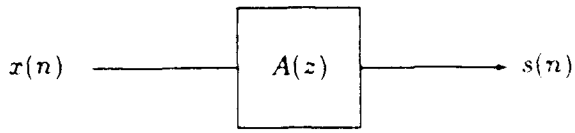


Figure 2.4: Filtering process to generate the all-zero residual sequence for the application of Durbin's method.

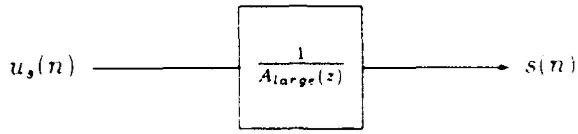


Figure 2.5: The residual sequence approximated by a large all-pole model.

pole model, $1/B(z)$, is then constructed for the sequence of *coefficients* in $A_{large}(z)$, the relationship obtained is

$$A_{large}(z) \approx \frac{1}{B(z)}. \tag{2.25}$$

Therefore the transfer function $1/A_{large}(z)$ of Figure 2.5 is replaced by

$$B(z) \approx \frac{1}{A_{large}(z)}, \tag{2.26}$$

which yields the desired moving average (all-zero) model.

c. Shank's Method

Consider the system shown in Figure 2.6 where $h_A(n)$ is the impulse response of the previously determined all-pole portion of a pole-zero model (using Prony's method for example). Shank's method [Ref. 42] is to satisfy the relationship of Figure 2.6 in the least squares sense. This relationship can be described by the matrix equation

$$\begin{bmatrix} h_A(Q) & h_A(Q-1) & \cdots & h_A(0) \\ h_A(Q+1) & h_A(Q) & \cdots & h_A(1) \\ \vdots & & & \vdots \\ h_A(N-1) & h_A(N-2) & \cdots & h_A(N-1-Q) \end{bmatrix} \begin{bmatrix} b_0 \\ b_1 \\ \vdots \\ b_Q \end{bmatrix}$$
$$= \begin{bmatrix} x(Q) \\ x(Q+1) \\ \vdots \\ x(N-1) \end{bmatrix} + \begin{bmatrix} \epsilon_B(Q) \\ \epsilon_B(Q+1) \\ \vdots \\ \epsilon_B(N-1) \end{bmatrix}. \tag{2.27}$$

or

$$\mathbf{H}_A \mathbf{b} = \mathbf{x} + \mathbf{e}_B. \tag{2.28}$$

Equation (2.28) can be solved in the manner of (2.6) with $\mathbf{H}_A$ analogous to $\mathbf{X}'_A$ and $\mathbf{x}$ analagous to $\mathbf{x}_A$. The b_k coefficients can therefore be found by using the psuedoinverse.

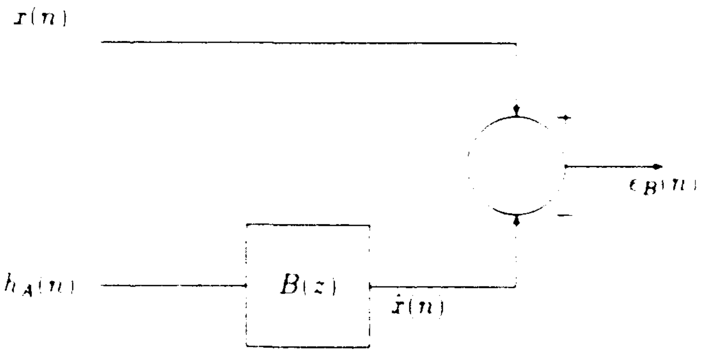


Figure 2.6: System for Shank's method determination of transfer function numerator coefficients.

3. Iterative Prefiltering

An iterative technique for solving the direct modeling problem of Figure 2.1 called *iterative prefiltering* has been proposed in [Ref. 24]. An effective application of

the technique has been reported in [Ref. 43]. The presentation below follows that of [Ref. 40].

In this method, the direct modeling problem error (see Figure 2.1a).

$$e_d(n) = x(n) - h(n) \tag{2.29}$$

is expressed in the alternative form

$$\begin{aligned} e_d(n) &= x(n) - b(n) * h_A(n) \\ &= x(n) * h_A(n) * a(n) - b(n) * h_A(n) \end{aligned} \tag{2.30}$$

where $a(n)$ and $b(n)$ are the sequences of transfer function denominator and numerator coefficients, respectively, and $h_A(n)$ is the impulse response of the AR (all-pole) portion of the model, i.e.

$$h_A(n) \Longleftrightarrow \frac{1}{A(z)}$$

By then making the equation error iterative (superscripts represent the index of iteration),

$$e_d^{i+1}(n) = x(n) * h_A^i(n) * a^{i+1}(n) - b^{i+1}(n) * h_A^i(n), \tag{2.31}$$

the least squares error solution for $a^{i+1}(n)$ and $b^{i+1}(n)$ at each iteration can be calculated using parameter estimates from the previous iteration. In matrix form (2.31) becomes

$$\begin{bmatrix} x_h(P) & \cdots & x_h^i(0) & h_A^i(P) & \cdots & h_A^i(P-Q) \\ x_h(P+1) & \cdots & x_h^i(1) & h_A^i(P+1) & \cdots & h_A^i(P-Q+1) \\ \vdots & \vdots & \vdots & \vdots & & \vdots \\ x_h(N-1) & \cdots & x_h^i(N-1-P) & h_A^i(N-1) & \cdots & h_A^i(N-1-Q) \end{bmatrix} \begin{bmatrix} 1 \\ a_1^{i+1} \\ \vdots \\ a_P^{i+1} \\ b_0^{i+1} \\ \vdots \\ b_Q^{i+1} \end{bmatrix}$$

$$= \begin{bmatrix} e_d^{i+1}(P) \\ e_d^{i+1}(P+1) \\ \vdots \\ e_d^{i+1}(N-1) \end{bmatrix} \tag{2.32}$$

where $x_h(n) = x(n) * h_A^i(n)$. Note that (2.32) can be solved in exactly the same manner as (2.6).

a. Correlation Domain Iterative Prefiltering

Many pole-zero modeling algorithm's which were originally conceived based of the time domain pole-zero difference equation, (2.1), have been reformulated in the correlation domain. Examples of this include the correlation domain extension of Prony's method resulting in the modified Yule-Walker equation methods, and an instrumental variable method of least squares system identification which Soderstrom has indicated is simply a correlation domain formulation of least squares system identification [Ref. 44]. In modeling trials conducted for this thesis both of these correlation domain methods were found to be significant improvements over their time domain counterparts.

Iterative prefiltering can also be extended into the correlation domain. To see how this is done first note that the direct formulation of the pole-zero modeling problem of Figure (2.1a) may be reinterpreted in the correlation domain by employing the relationship of (2.20) and Figure 2.3. Figure 2.7 illustrates this new direct pole-zero modeling interpretation.

Proceeding as for the time domain iterative prefiltering above we write the correlation domain error equation,

$$\begin{aligned} e_{d,corr}(n) &= \hat{r}_{xx}(n) - x(-n) * h(n) \\ &= \hat{r}_{xx}(n) * h_A(n) * a(n) - x(-n) * b(n) * h_A(n) \end{aligned} \tag{2.33}$$

where, as before, $a(n)$ and $b(n)$ are sequences of the model coefficients, $h_A(n)$ is the impulse response of the AR (all-pole) portion of the estimated model, and $\hat{r}_{xx}(n)$ is found using $x(n)$ as the desired impulse response so that $\hat{r}_{xx}(n) = x(n) * x(-n)$.

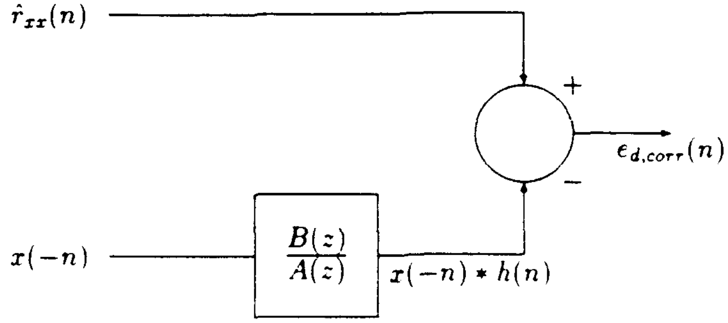


Figure 2.7: The direct pole-zero modeling problem formulation expressed in the correlation domain.

When the error is made iterative, (2.33) becomes

$$\epsilon^{i+1}_{d,corr}(n) = x(n) * x(-n) * h^{i}_{A}(n) * a^{i+1}(n) - x(-n) * b^{i+1}(n) * h^{i}_{A}(n) \qquad (2.34)$$

where the superscripts represent the index of iteration. In matrix form (2.34) becomes

$$\begin{bmatrix} r_h(P) & \cdots & r^{i}_{h}(0) & x^{i}_{A}(P) & \cdots & x^{i}_{A}(P-Q) \\ r_h(P+1) & \cdots & r^{i}_{h}(1) & x^{i}_{A}(P+1) & \cdots & x^{i}_{A}(P-Q+1) \\ \vdots & \vdots & \vdots & \vdots & & \vdots \\ r_h(2N-1) & \cdots & r^{i}_{h}(2N-1-P) & x^{i}_{A}(2N-1) & \cdots & x^{i}_{A}(2N-1-Q) \end{bmatrix} \begin{bmatrix} 1 \\ a^{i+1}_{1} \\ \vdots \\ a^{i+1}_{P} \\ b^{i+1}_{0} \\ \vdots \\ b^{i+1}_{Q} \end{bmatrix}$$

$$= \begin{bmatrix} \epsilon^{i+1}_{d,corr}(P) \\ \epsilon^{i+1}_{d,corr}(P+1) \\ \vdots \\ \epsilon^{i+1}_{d,corr}(N-1) \end{bmatrix} \qquad (2.35)$$

where $r_h(n) = \hat{r}_{xx}(n) * h^{i}_{A}(n)$ and $x_h(n) = x(-n) * h^{i}_{A}(n)$ and which can be solved as before.

4. Maximum Likelihood Techniques

Maximum likelihood estimation of parameters is the statistical standard against which the performance of other estimators is measured. This estimator makes use of all the useful statistical information available in a given set of data [Ref. 45, p. 73]. Difficulties arise, however, in the implementation of the maximum likelihood estimator (MLE). True maximum likelihood estimation requires exact knowledge of the conditional probability density function (PDF) of the observed data conditioned on the parameters to be estimated. This conditional PDF must then be simultaneously maximized with respect to all parameters being estimated. In practice, most efforts to employ maximum likelihood estimation make simplifying assumptions about the nature of the input data to derive a useful algorithm. Such techniques are usually called *approximate* maximum likelihood methods.

The approximate MLE chosen for this thesis is due to Akaike [Ref. 31]. The brief developement of this algorithm provided below follows that of Kay [Ref. 4, Ch. 9,10]. Additional backround on maximum likelihood estimation can be found in [Ref. 1, 8, 32, 45] and references therein.

Given a sequence of independent random variable observations, $x(n)$, and a corresponding set of parameters to be estimated, θ_k, the desired set of estimates for the θ_k's is the one for which the observed data set is the *most likely*. In terms of the conditional probability density or likelihood function,

$$p(x(0), x(1), \cdots, x(N-1)|\theta_1, \theta_2, \cdots, \theta_K),$$

the desired set of estimates is the one which, for a given set of $x(n)$, is maximized. In the case of pole-zero modeling, an expression for the observed data sequence's joint probability density function conditioned on the model parameters, a_i and b_j, is required. To obtain such an expression, it is generally assumed that the observed data

is Gaussian. If, further, it is assumed that the input to the pole zero model is white Gaussian noise and that the data record length is much longer than the transient response due to initial conditions, the conditional PDF for the observed data can be arrived at fairly directly.

The joint PDF for the zero mean white Gaussian input sequence, $u(n)$, is of the form

$$p(u(0), u(1), \cdots, u(N-1)) = \prod_{i=0}^{N-1} \frac{1}{\sqrt{2\pi\sigma_u^2}} exp(-\frac{u^2(i)}{2\sigma_u^2}). \tag{2.36}$$

where σ_u^2 is the variance. Now the density function for $x(0), x(1), \cdots, x(N-1)$ conditioned on the θ_k's, can be found from (2.36) through the standard linear transformation,

$$p(x(0), x(1), \cdots, x(N-1)) = p(u_f(0), u_f(1), \cdots, u_f(N-1)) |\mathcal{J}|, \tag{2.37}$$

where $u_f(n)$ is the inverse filter relationship of the original pole-zero difference equation, (2.1). Specifically

$$u_f(n) = \frac{1}{b_0} \sum_{i=0}^{P} a_i x(n-i) - \frac{1}{b_0} \sum_{k=1}^{Q} b_k u(n-k) \tag{2.38}$$

and $\mathcal{J}$ is the Jacobian of the linear transformation u_f. To simplify the transformation assume that the pole-zero filter in (2.38) has been expressed as its equivalent all-pole filter,

$$u(n) = \sum_{i=0}^{P_{AP}} a_{AP,i} x(n-i). \tag{2.39}$$

The final result of the linear transformation (2.37) will then be

$$p(x(0), x(1), \cdots, x(N-1)) = \prod_{n=0}^{N-1} \frac{1}{\sqrt{2\pi\sigma_u^2}} exp(-\frac{\sum_{k=0}^{P_{AP}} a_{AP,i} x(n-k)}{2\sigma_u^2}). \tag{2.40}$$

When (2.40) is expressed in terms of the pole-zero parameters,

$$a_1, a_2, \cdots, a_P, b_0, b_1, \cdots, b_Q,$$

for the purpose of maximization the relationship is highly nonlinear. Note that the maximization of (2.40) requires the minimization, over all possible a_i's and b_k's, of the inverse filter error, $u_f(n)$.

Akaike employed the Newton-Raphson iterative method for minimizing (2.40). This method requires the computation of Gradient and the Hessian of (2.38) at each iteration to generate the estimate updates

$$\begin{bmatrix} \mathbf{a}_{k+1} \\ \mathbf{b}_{k+1} \end{bmatrix} = \begin{bmatrix} \mathbf{a}_k \\ \mathbf{b}_k \end{bmatrix} + \begin{bmatrix} \frac{\partial^2 Q}{\partial \mathbf{a} \partial \mathbf{a}^T} & \frac{\partial^2 Q}{\partial \mathbf{a} \partial \mathbf{b}^T} \\ \frac{\partial^2 Q}{\partial \mathbf{b} \partial \mathbf{a}^T} & \frac{\partial^2 Q}{\partial \mathbf{b} \partial \mathbf{b}^T} \end{bmatrix} \begin{bmatrix} \frac{\partial^2 Q}{\partial \mathbf{a}} \\ \frac{\partial^2 Q}{\partial \mathbf{b}} \end{bmatrix}. \tag{2.41}$$

Using frequency domain arguments, Akaike was able to provide expressions for the above partial derivatives in terms of Fourier transforms which can in turn be expressed in terms of linear filtering operations.

Note that there are several key assumptions made above which must be valid for this method of approximate MLE to apply:

1. The data are real, Gaussian, and zero mean.

2. The data record is large. This is to avoid end effects of assuming all data values outside the data record are zero when filtering the data.

3. The poles and zeros are not close to the unit circle. This is to avoid long transients due to the initial conditions which are ignored. (They are assumed to be known and are set equal to zero.)

At first glance these assumptions would seem to indicate that this method is inappropriate to transient modeling. However, inverse filter error retains its meaning when considering transient waveforms; the ideal inverse filtering result for a transient signal is a single impulse rather than the minimum variance random sequence expected for a stochastic process. In fact, this is exactly the appproach taken by Jackson [Ref. 26, pp. 276-278] in extending Judell's maximum likelihood method [Ref. 33] to impulse

response data. While this extension seems rather *ad hoc*, we will find that such approximate maximum likelihood methods can be effective in modeling transient signals as impulse responses. A key limitation imposed by dealing with deterministic data is that reliance on inverse filter error excludes signals that must be modeled by non-minimum phase systems. In contrast, the restrictions of long data record and weak poles and zeros (not near the unit circle) no longer apply. Data records end effects and initial condition transient effects should have no impact since the assumption of zero valued data outside the range of data is correct if the data is chosen to end after most of the energy of the transient is dissipated.

C. IMPLEMENTATION

All modeling algorithms were implemented using the interactive language PRO-MATLAB from *The Mathworks, Inc.* on SUN workstations except for the Akaike MLE algorithm which was implemented in FORTRAN using a program adapted from [Ref. 4, Ch. 10]. The FORTRAN program was also implemented on a SUN workstation with a FORTRAN 77 compiler All graphics were generated in PRO-MATLAB.

III. MODELING PERFORMANCE

A. PERFORMANCE CONSIDERATIONS

Each of the pole-zero modeling techniques presented in Chapter II is effective when modeling a signal that is truly the impulse response of a pole-zero system with no noise present and with the system order known. However, real world transient data rarely possess such characteristics. Real world signals of all types are notoriously uncooperative in fitting the signal models proposed to describe them. Reasons that this may be true for transient data include:

1. Inappropriate selection of model type or modeling algorithm.
    - Linear versus non-linear models.
    - Minimum phase versus non-minimum phase rational models.
2. The transient is time shifted because of and inappropriate selection of the data record starting point due to the presence of noise.
3. The assumption of impulse system excitation is a poor approximation.
4. Incorrect selection of model order.
5. Noise is present in the signal.

The test sequences used in this chapter are all obtained as the impulse response of linear pole-zero systems, therefore, the question of linear versus non-linear model type will not be at issue. For the other problems, effective transient modeling requires both selecting the appropriate algorithm and understanding how to use that algorithm to its greatest advantage. The next section describes the test sequences used in this

chapter. Subsequent sections address how difficulties encountered in the pole-zero modeling of impulse response data can arise and how they can be overcome.

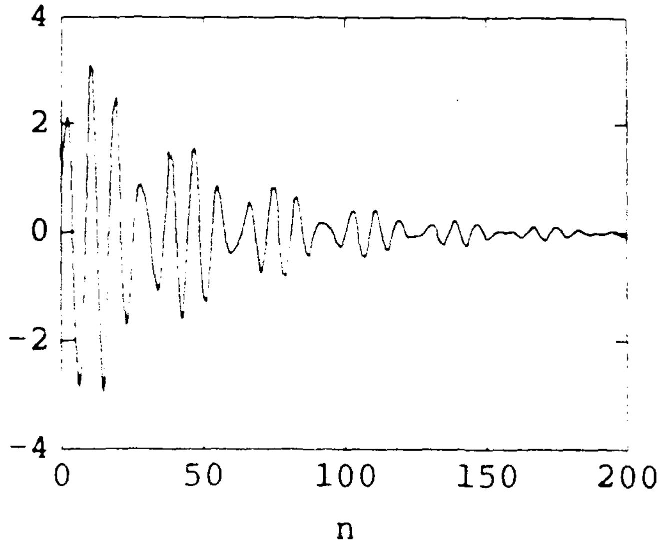
The effects of data selection, non-minimum phase systems, non-impulse excitation, and incorrect model order on modeling will initially be considered for signals observed with no added noise present. The effect of modeling a signal in which additive noise is present is considered separately. We will see that situations which modify a linear pole-zero system often lead to another pole-zero system. This new system usually has the same number of poles in the same locations but with different and possibly additional zeros.

B. TEST SEQUENCES

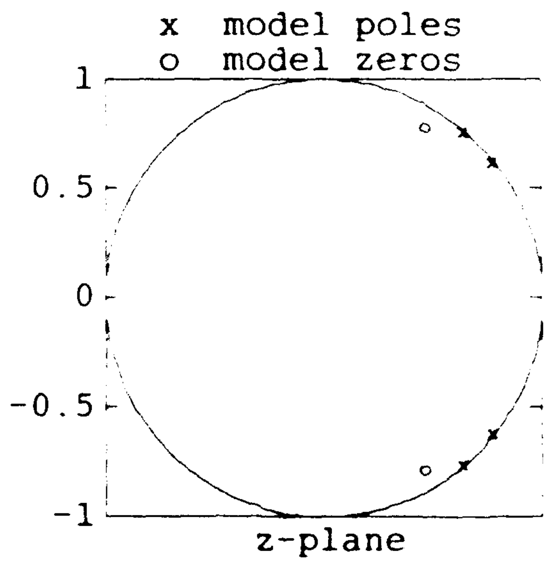
The test impulse response sequences are generated using pole-zero models taken from Kay [Ref. 4]. The test sequence ARMA3 uses one of Kay's models directly while the test sequences ARMA4 LF, ARMA3 NM, and ARMA4 CL are from Kay models which have been modified to enhance the illustration of certain points. The unit impulse response and pole-zero plot of each test sequence model is shown in Figures 3.1a-h. The model coefficients for these sequences are listed in Table 3.1.

Model	Model Coefficients ($a_0 = b_0 = 1$)					
	a_1	a_2	a_3	a_4	b_1	b_2
ARMA3	-2.760	3.809	-2.654	0.924	-0.900	0.810
ARMA4 LF	-3.035	4.002	-2.727	0.778	-0.200	0.040
ARMA3 NM	-2.760	3.809	-2.654	0.924	-4.818	25.000
ARMA4 CL	-2.678	3.700	-2.5634	0.917	-0.200	0.040

TABLE 3.1: Table of test sequence coefficients.

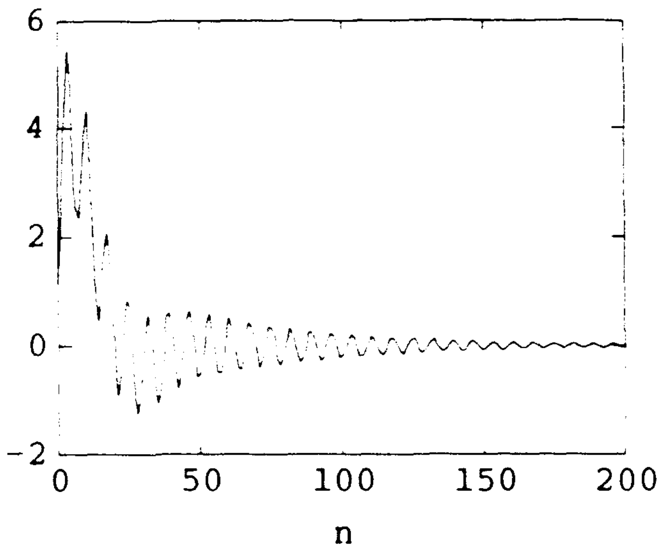


(a)

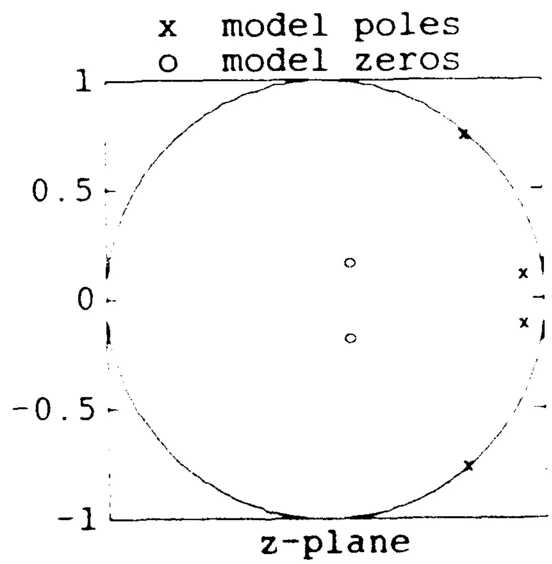


(b)

Figure 3.1: The test sequence ARMA3. (a) Impulse response plot and (b) pole-zero plot.

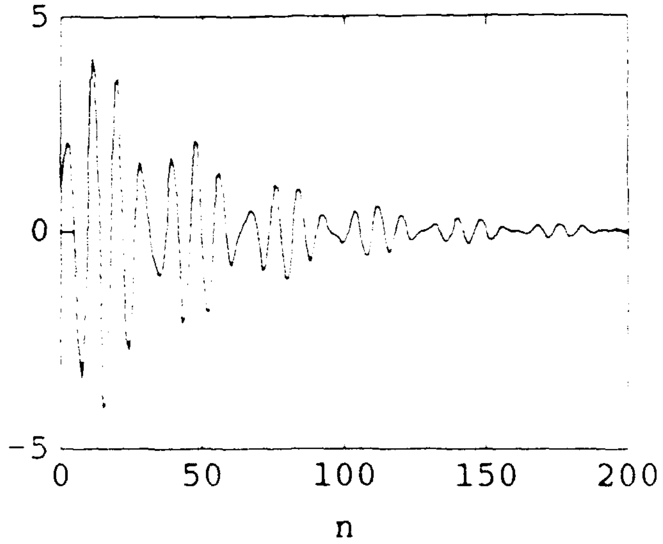


(c)

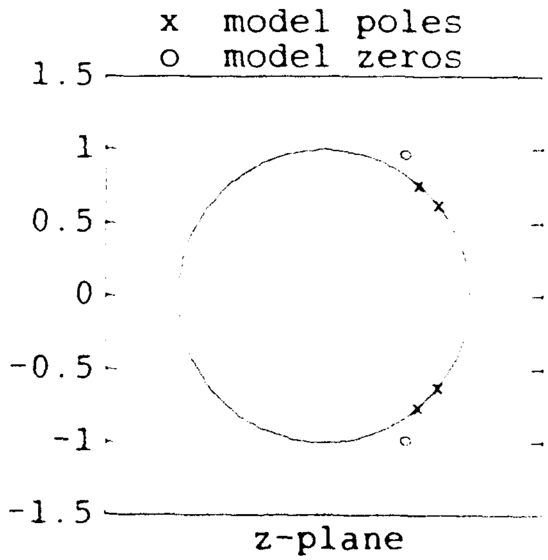


(d)

Figure 3.1: continued The test sequence ARMA4 LF. (c) Impulse response plot and (d) pole-zero plot.

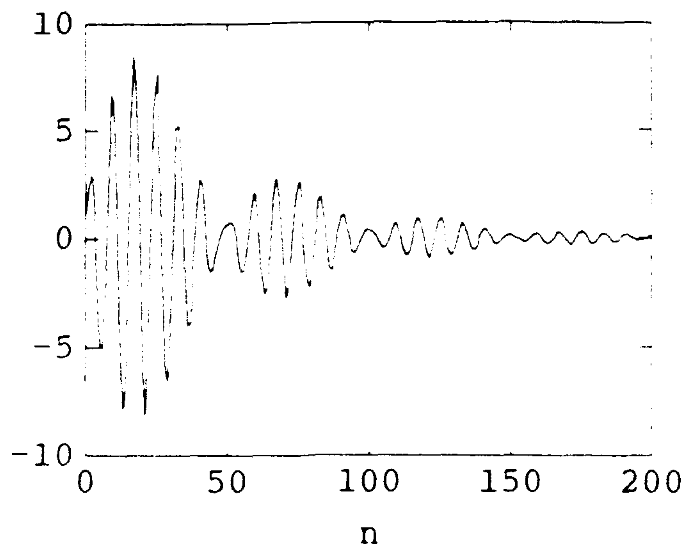


(e)

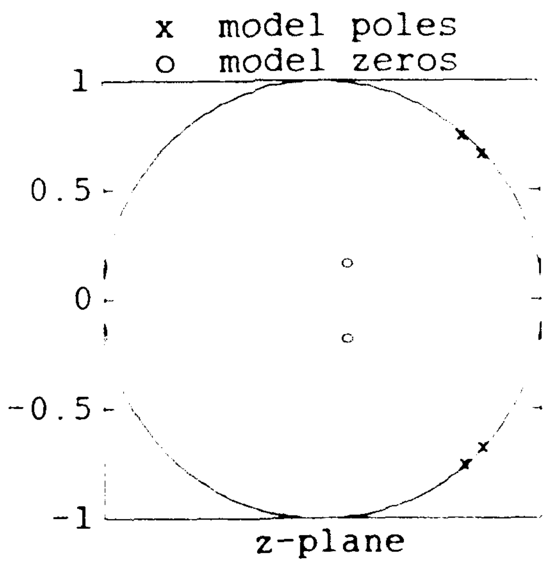


(f)

Figure 3.1: continued The test sequence ARMA3 NM. (e) Impulse response plot and (f) pole-zero plot.



(g)



(h)

Figure 3.1: continued The test sequence ARMA4 CL. (g) Impulse response plot and (h) pole-zero plot.

C. ESTIMATING NUMERATOR COEFFICIENTS

1. Data Selection—Time Shifts and Initial Conditions

Defining the range of data to be used in modeling is an important and usually staightforward exercise. The assumption that a signal represents the impulse response of a linear pole-zero system, however, implies some very specific properties about the initial few points of that signal. Each method of modeling the transfer function numerator coefficients in Chapter II reacts differently when the beginning points of the impulse response being modeled are degraded. Because real world transients do not usually exhibit the instantaneous rise time of an ideal impulse response and because noise is usually present, choosing the precise data range for a transient such as that illustrated in Figure 3.2 is often a very uncertain task. Two possible outcomes when the transient starting point is chosen incorrectly are:

1. The starting point is chosen before the signal begins so that early data values are unrelated (and presumably of lower amplitude) to the impulse response to be matched (e.g. these points may consist of noise).

2. The starting point is chosen late in which case early values of the impulse response are lost.

In the first case, an adequate number of additional model zeros can account for the delay in the impulse response. Assuming that the starting point for the data is reasonably close to the true beginning of the impulse response, any spectral features introduced by the unrelated early data points will probably not have enough energy to significantly alter the spectrum of the impulse response. Under these circumstances all methods can effectively find the poles. It is important to note, however, that if an insufficient number of zeros to account for the imposed delay is used, then some of the equations that are generated in Prony's method become invalid. When these

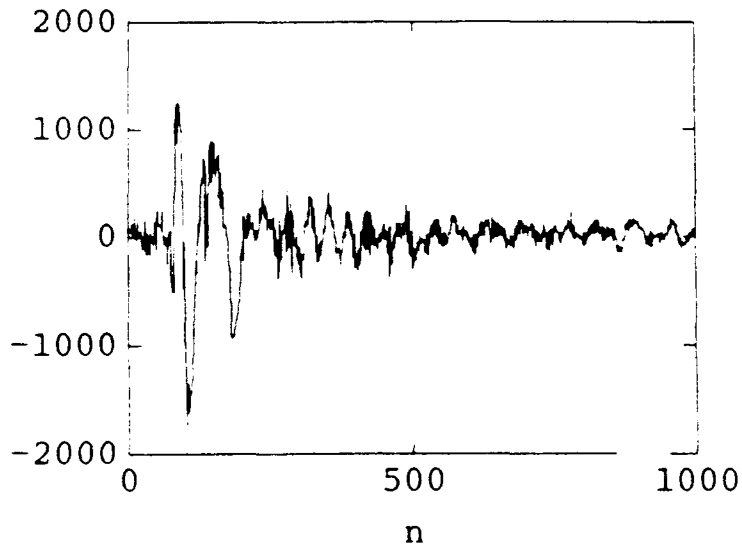


Figure 3.2: An example of a laboratory generated acoustic transient. Note the difficulty in determining a precise starting point for the transient.

equations are solved, the invalid equations can drastically degrade pole estimates. To see this we can apply Prony's method to a system with the true orders $P = 4$ and $Q = 2$. Assume the signal is delayed by inserting three zeros at the beginning of the data so that the original point $x(0)$ is now the fourth data point. If we choose $P = 4$ and $Q = 3$ in constructing the data matrix of (2.2), the resulting set of equations will be

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ x(0) & 0 & 0 & 0 & 0 \\ x(1) & x(0) & 0 & 0 & 0 \\ x(2) & x(1) & x(0) & 0 & 0 \\ x(3) & x(2) & x(1) & x(0) & 0 \\ x(4) & x(3) & x(2) & x(1) & x(0) \\ \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix} \begin{bmatrix} 1 \\ a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} = \begin{bmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \\ 0 \\ \vdots \end{bmatrix}$$

Note that when the lower partition is taken to find the a_k coefficients, the first two equations of the lower partition are invalid. Compounding the problem is that the

invalid equations occur in the high energy portion of a transient signal. To overcome this effect a numerator order of at least five is required.

In finding the numerator coefficients of a delayed impulse response one of three outcomes is possible depending on the estimation technique chosen. First, any technique which depends directly on the initial values of the data sequence (for example (2.5)) will be ineffective. Second, methods which rely on the autocorrelation of the residual sequence will result in approximately the true zeros of the system under study. The original time series will not be matched directly. This case is illustrated in Figure 3.3. The resulting impulse response is an undelayed version of the signal. Finally, signal matching techniques, iterative prefiltering and Shank's method, result in zeros not related to the original undelayed model but which provide the best overall time domain match of the delayed signal. This is shown in Figure 3.4.

The case of choosing the data record to far to the right and thus truncating the first points of an impulse response will again have little effect on pole estimation. This situation corresponds the same system with (non-zero) initial conditions imposed. Since initial conditions are acounted for in the numerator, the zeros are significantly altered. Here the previous discussion regarding finding the underlying model zeros versus obtaining a good match in the time domain still applies with one exception: the direct calculation of the b_k coefficients from (2.5) will now be effective.

2. Non-minimum Phase Modeling

In [Ref. 46] it is demonstrated that the appropriate discrete model of a sampled analog waveform is often represented by a transfer function with zeros outside the unit circle. Many of the techniques that are currently available are purposefully structured to *eliminate* uch non-minimum phase models. In power spectrum estimation, a minimum phase system with the same frequency response magnitude

as a non-minimum phase system results in the same estimated spectrum. Thus for spectrum estimation an equivalent minimum phase system is satisfactory. In fact, the assumption underlying stochastic modeling techniques, namely white Gaussian noise input, guarantees that all transfer function combinations of minimum phase and maximum phase zeros are equivalent. By convention, stochastic modeling techniques always choose the minimum phase model so that the important statistic of inverse filter error is available. In time domain based applications, however, incorrectly choosing the model phase can seriously degrade system performance. Fields such as seismic deconvolution, channel equilization, control, and matched filter design, generally require identification of the correct model phase [Ref. 47, 48, 49]. Also, we will see in Chapter IV that effective modeling of real world acoustic signals frequently requires non-minimum phase models.

A number of modifications to the basic stochastic model have been introduced to allow selection of the model with the correct phase. These techniques generally involve changing the Gaussian nature of the input noise [Ref. 50] and often employ higher than second order moments or cumulants [Ref. 29, 30]. However, when a model's impulse response has effectively matched a signal in the time domain, the resulting phase is immediately known to be the correct. Allowing for the possibility of non-minimum phase models places significant limitations on the modeling methods which may be used. Techniques which rely on inverse filtering (maximum liklihood methods) are not applicable since the inverse of a non-minimum phase system is unstable. Also, techniques which use correlation information to calculate the b_k coefficients (spectral factorization and Durbin's method) will give poor results since correlation data does not preserve phase information. Figure 3.5 shows the difficulty encountered when Durbin's method attempts to model a non-minimum phase system. The best Durbin's method can do is produce the spectrally equivalent minimum phase version

of a maximum phase system since the autoregressive modeling techniques on which it relys can only produce zeros within the unit circle. In contrast, Figure 3.6 shows that Shank's method is able to find the correct model.

3. Numerator Modeling Summary

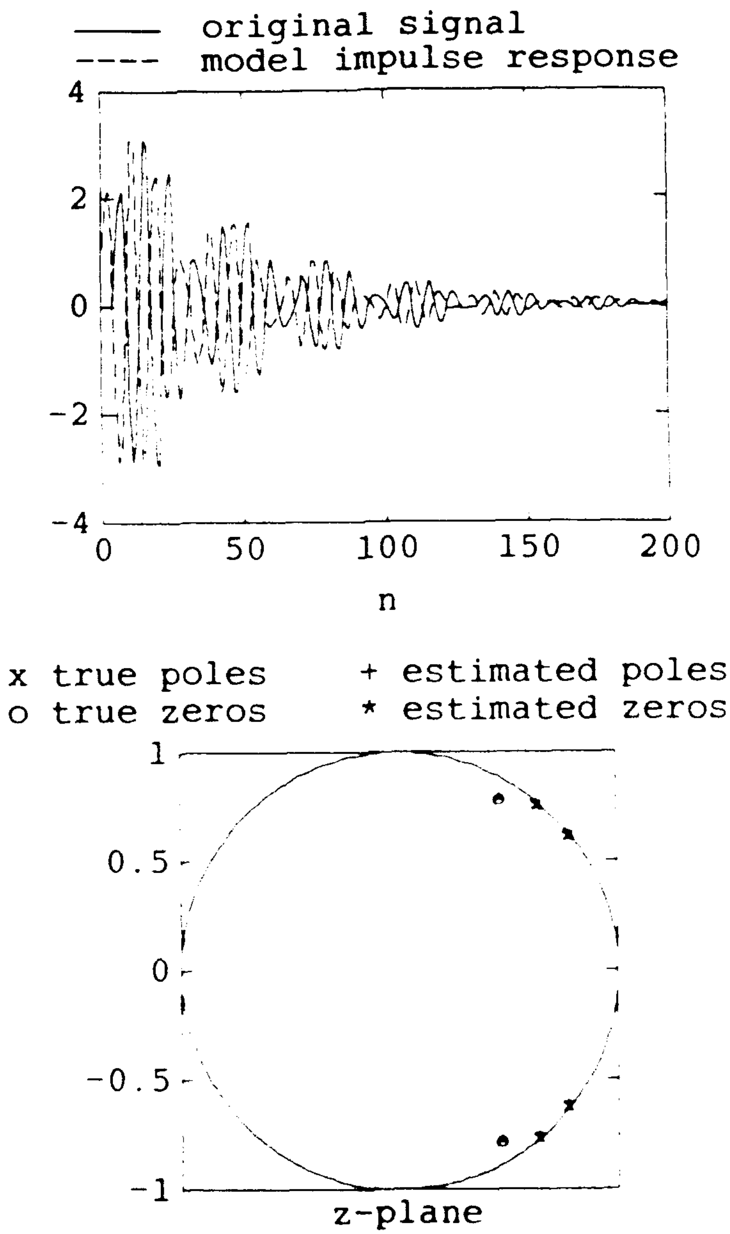
Table 3.2 provides a brief summary of the modeling properties of the numerator coefficient modeling techniques considered in this thesis. Since the goal in Chapter IV is to perform time domain modeling of the acoustic transients being considered, those methods which provide the best time domain match between the original signal and the model impulse response are preferred.

Method	Equation	Non-minimum phase capable?	Data Selection	
			Shifted Right (Delayed)	Shifted Left (Truncated)
Prony, upper partition	2.5	Yes	Not Usable	Time Series Match
Spectral Factorization	2.24	No	Underlying Model	Underlying Model
Durbin's Method	2.25 2.26	No	Underlying Model	Underlying Model
Shank's Method	2.28	Yes	Time Series Match	Time Series Match
Iterative Prefiltering	2.32	Yes	Time Series Match	Time Series Match
Akaike MLE	2.41	No	Underlying Model	Time Series Match

TABLE 3.2: Summary of the capabilities and limitations of numerator modeling methods.

D. NON-IMPULSE EXCITATION

In any real world system the assumption of a unit impulse input is approximate. If the duration of the excitation waveform is small relative to the period of the



(a)

(b)

Figure 3.3: The right shifted sequence ARMA3, poles modeled using LSMYWE and zeros modeled using Durbin's method. (a) Time signal plot and (b) pole-zero plot. Durbin's method does not account for the time delay but instead finds the underlying system's true zeros.

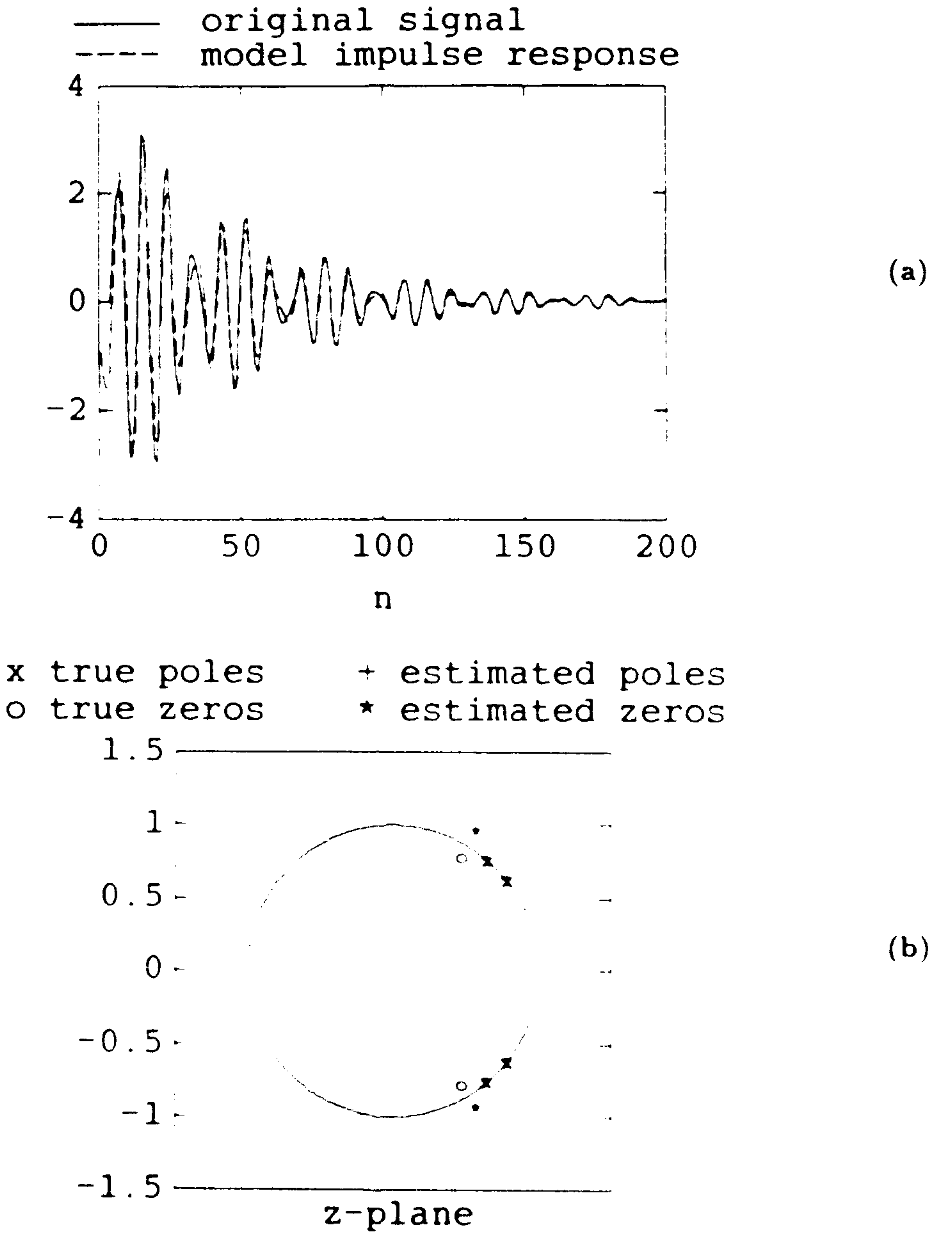


Figure 3.4: The right shifted sequence ARMA3, poles modeled using LSMYWE and zeros modeled using Shank's method. (a) Time signal plot and (b) pole-zero plot. The least squares method does not find the underlying system's true zeros but rather achieves the best overall time domain match.

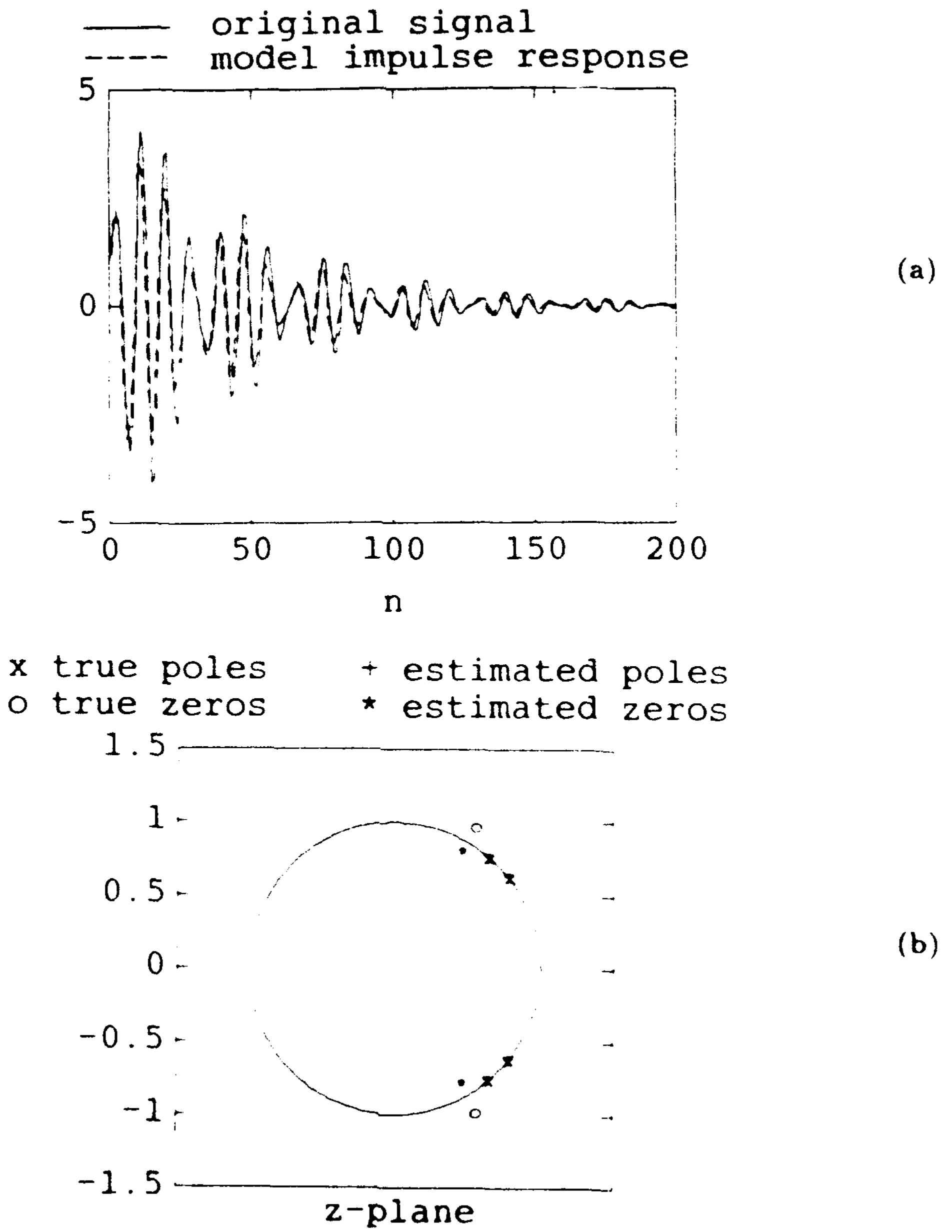
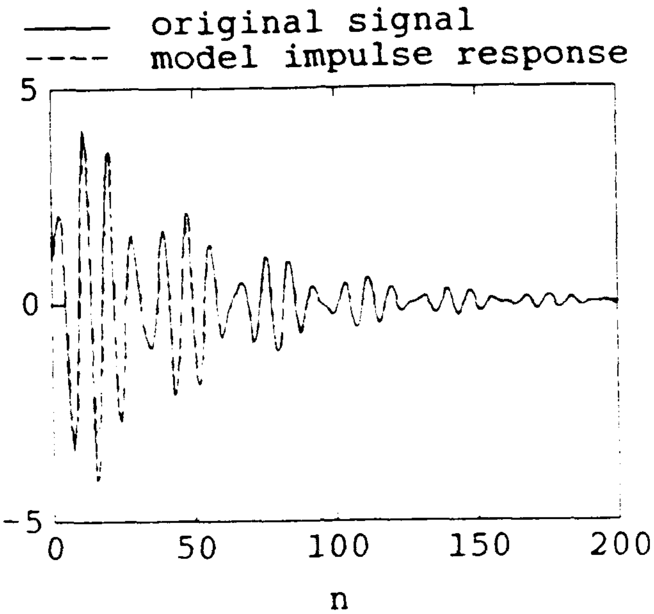
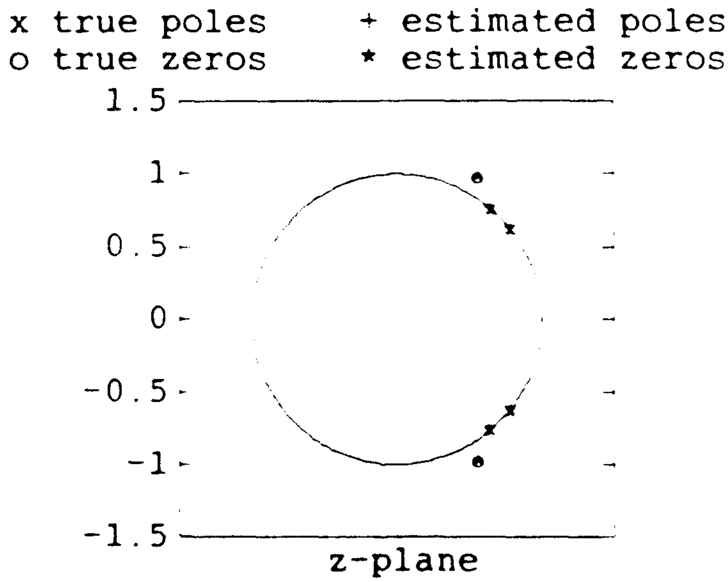


Figure 3.5: The non-minimum phase sequence ARMA3 NM, poles modeled using LSMYWE and zeros modeled using Durbin's method. (a) Time signal plot and (b) pole-zero plot. Durbin's method cannot model zeros outside the unit circle.



(a)



(b)

Figure 3.6. The non-minimum phase sequence ARMA3 NM, poles modeled using LSMYWE and zeros modeled using Shank's method. (a) Time signal plot and (b) pole-zero plot. The least squares method is effective at modeling zeros outside the unit circle.

lowest oscillation frequencies present then the assumption is justified. However, it is reasonable to expect this criterion will often not be satisfied. Non-impulse excitations which may be encountered include:

1. A long duration baseband type pulse.

2. An uncorrelated random train of impulses. This model is often used to account for reverberation (echoes) in siesmic deconvolution [Ref. 51].

3. A frequency swept input that sweeps through the natural frequencies of a system. This model is usually considered in conjunction with the starting and stopping of rotating machinery.

If the system input were known, the modeling problem could be formulated as a system identification problem. When no information about the system input is available, other means must be found to deal with this problem.

1. Baseband Pulse Excitation

The effect of modeling a transient signal from a linear system in which the input is a long duration, baseband-type pulse can best be understood as filtering by a finite impulse response (FIR, all-zero) filter as illustrated in Figure 3.7. The spectral properties of the original time series are windowed by the frequency response of the FIR filter coefficients. For this type of pulse, the effect is that of low pass filtering. Thus high frequencies are attenuated relative to low frequencies. If no frequency component exists below the FIR filter's cutoff frequency then the original spectrum is altered according to the side lobe structure of the FIR filter.

The new model that results can be viewed as a system with the original model poles but with new numerator polynomial coefficients that are the result of convolving the FIR filter coefficients with the original numerator polynomial coefficients. This results in a higher order polynomial, hence more zeros than were present

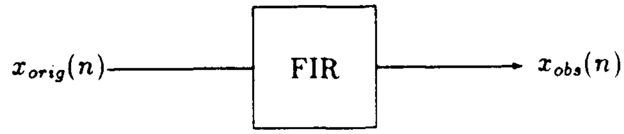


Figure 3.7: One way to view a linear system excited by a baseband pulse.

in the original model will be required. Observe how the original impulse response of Figure 3.7a is altered to Figure 3.7b when a nine element triangular excitation pulse is used. The dotted lines in Figures 3.7b,c,d indicate the modeling results obtained when none, two, and four extra zeros, respectively, are used in the estimated pole-zero model. In this case four extra zeros prove sufficient to account for the input pulse. The corresponding model spectrum, Figure 3.7e illustrates the attenuation caused by the baseband excitation. The pole-zero plot in Figure 3.7f shows that non-minimum phase zeros were required to achieve an effective time domain match.

2. Random Impulse Train Excitation

If the input to a linear system is an uncorrelated random train of impulses then, although the time series may be significantly different from the original impulse response, the autocorrelation function of the signal is theoretically unaltered except for a scaling factor. This is because the autocorrelation of an uncorrelated impulse train is a scaled unit impulse. Thus modeling methods which rely on correlation information should be effective. However, over a finite time interval it is unlikely that a random sequence will be truly uncorrelated. As the impulse train becomes correlated the situation will be equivalent to the baseband pulse case described above.

3. Frequency Swept Excitation

The output of a system excited with a frequency swept signal depends on the rate at which the sweep occurs. A slow sweep will result in a series of transient events, each at a specific resonant frequency of the system. These events can each

be modeled separately. When the sweep rate is rapid, all natural modes will appear much as if the input were an impulse except that the phase relationships of the various components may be changed.

E. MODEL ORDER SELECTION

In studies of rational modeling the issue which continues to be the most confounding is that of model order selection. The proposed methods which have a sound theoretical basis (e.g. [Ref. 52, 53, 54]) are very difficult to actually implement. These methods are invariably related to maximum likelihood concepts and therefore rely heavily on inverse filter statistics. This implies that for model order evaluation the inverse filter error must be calculated over all possible model orders. Then the model order and inverse filter error which minimize some function of the two is selected. The case of non-minimum phase systems is even more intractable since the inverse filter of such systems is unstable.

A more attractive but less understood method is to initially overdetermine a system and then allow the modeling algorithm to indicate the correct model order. A method proposed by Cadzow [Ref. 39] uses singular value decomposition to aid in model order selection in the denominator of all-pole and pole-zero models. Kumeresan and Tufts [Ref. 21] have shown that when Prony's method is applied to exponentially damped sinusoids reversed in time, valid poles occur outside the unit circle and excess poles occur inside the unit circle. In both of these methods the denominator order is initially overdetermined and the modeling method then provides the correct order. For this thesis both of these methods were applied to a number of transients. They were effective when applied to the noiseless impulse responses of true pole-zero systems but were not robust in the presence of noise or when many narrowband components are present. There is no similar guidance for determining the numerator model order.

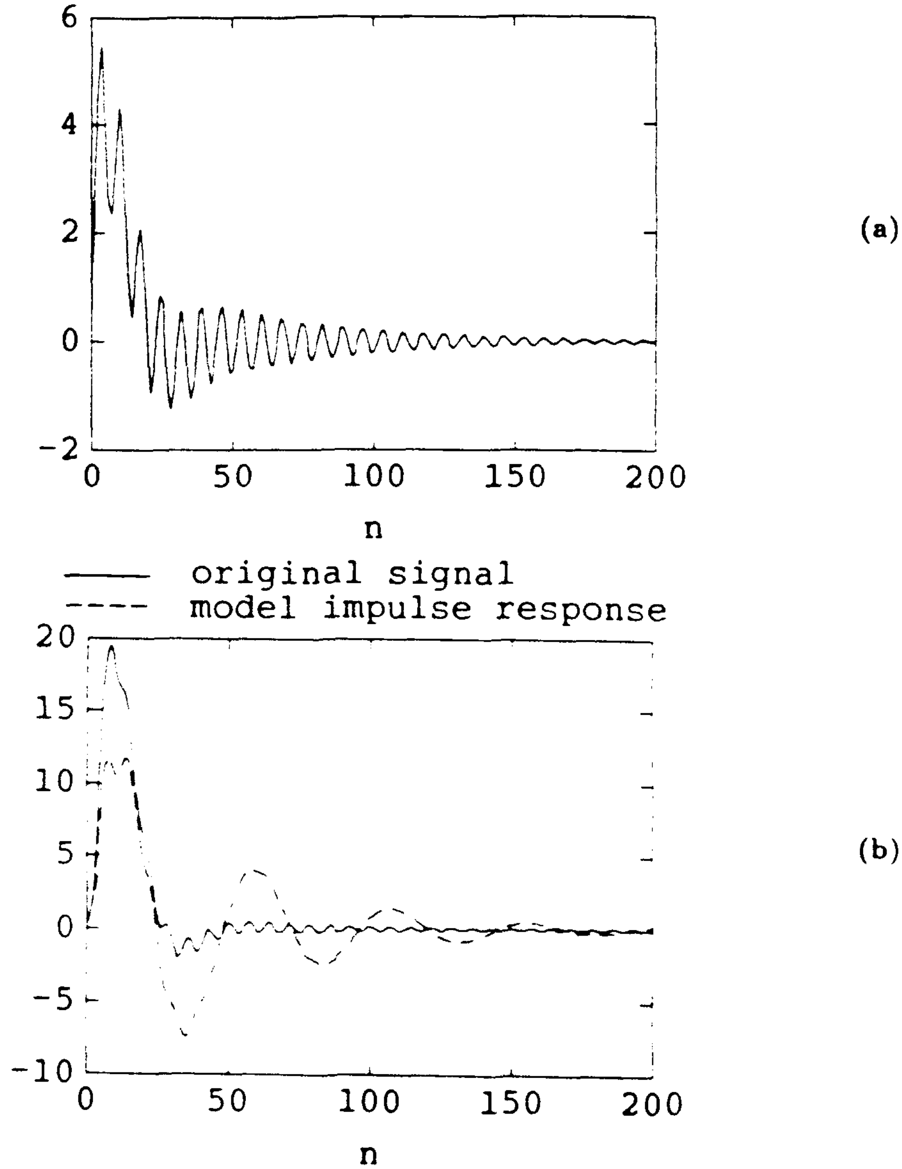
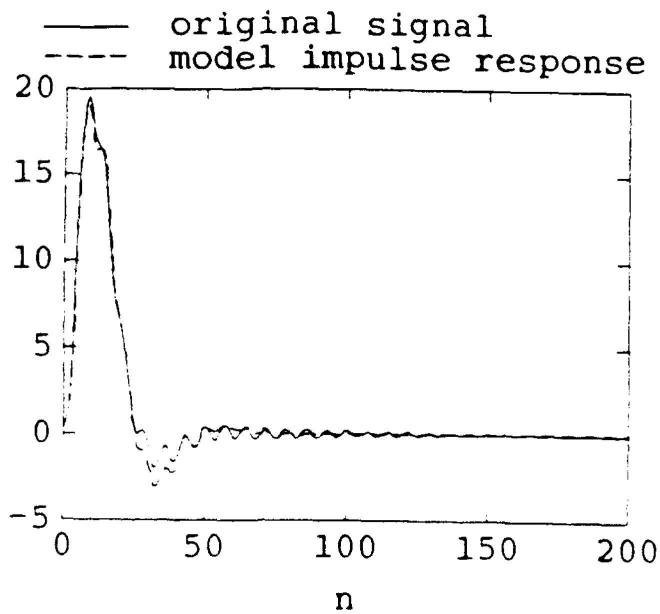
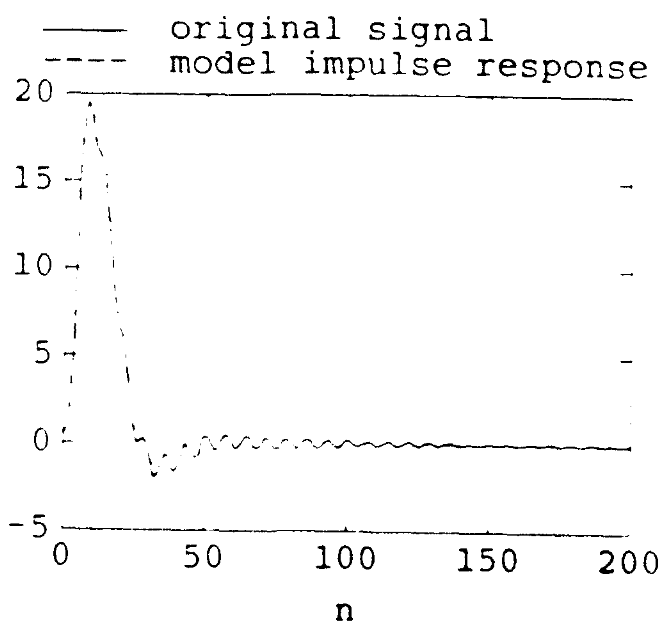


Figure 3.8: The sequence ARMA4 LF with (a) unit impulse excitation and (b) triangular pulse input modeled with correct model orders $P = 4$ and $Q = 2$ using Prony's method and Shank's method.

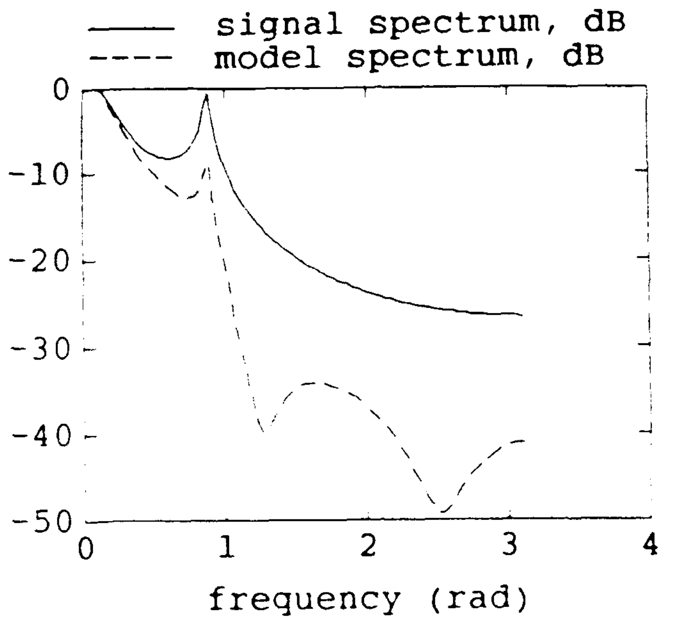


(c)

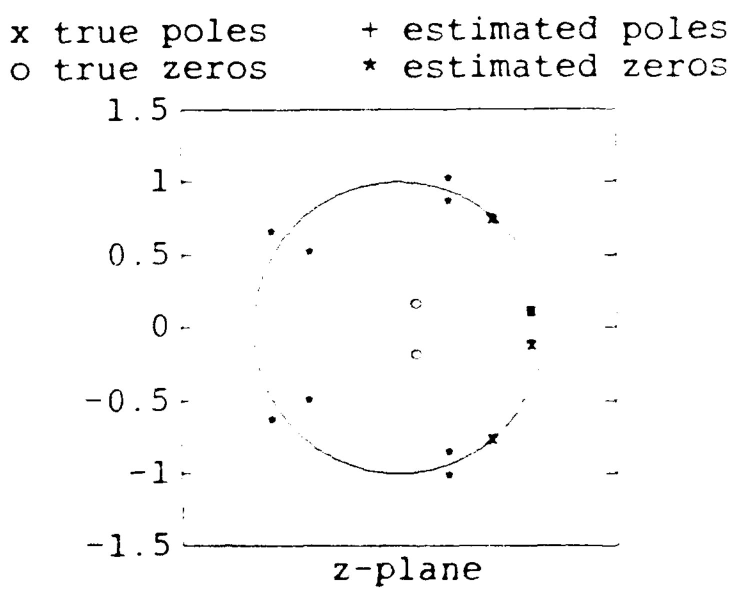


(d)

Figure 3.8: continued The sequence ARMA4 LF with triangular pulse input (c) modeled with Prony's method and least squares identification with orders $P = 4$ and $Q = 4$ (d) modeled with Prony's method and Shank's method orders $P = 4$ and $Q = 6$.



(e)



(f)

Figure 3.8: continued The sequence ARMA4 LF with triangular input (e) the original impulse response and triangular output model, order (4,6), spectrums and (f) the corresponding pole-zero plot. Note how the base-band pulse has attenuated the higher frequencies.

Perhaps most desirable would be a modeling technique that, when overdetermined, caused excess poles and zeros to either cancel or migrate well away from the unit circle (all the way to the origin ideally). Tummala [Ref. 23] has demonstrated some success with this concept using an iterative algorithm to solve the least squares identification problem. Observing the degree to which the various transient modeling techniques of this thesis exhibit this behavior is the approach taken in the following comparison. This behavior was observed by modeling the ARMA3 test sequence with different combinations of numerator and denominator order. Table 3.3 summarizes the results obtained. A 'Y' in Table 3.3 indicates a modeling technique achived an exact time domain match between the model impulse response the modeled signal. The most notable negative result is that both Prony's method and LSMYWE were ineffective when the numerator *and* denominator were overdetermined. Also, excess poles without enough zeros causes problems for correlation domain iterative prefiltering. Figure 3.9 is an example of the results obtained using LSMYWE when excess poles and zeros are present. However, when these results were used to initialize iterative prefiltering the excess poles and zeros were handled effectively.

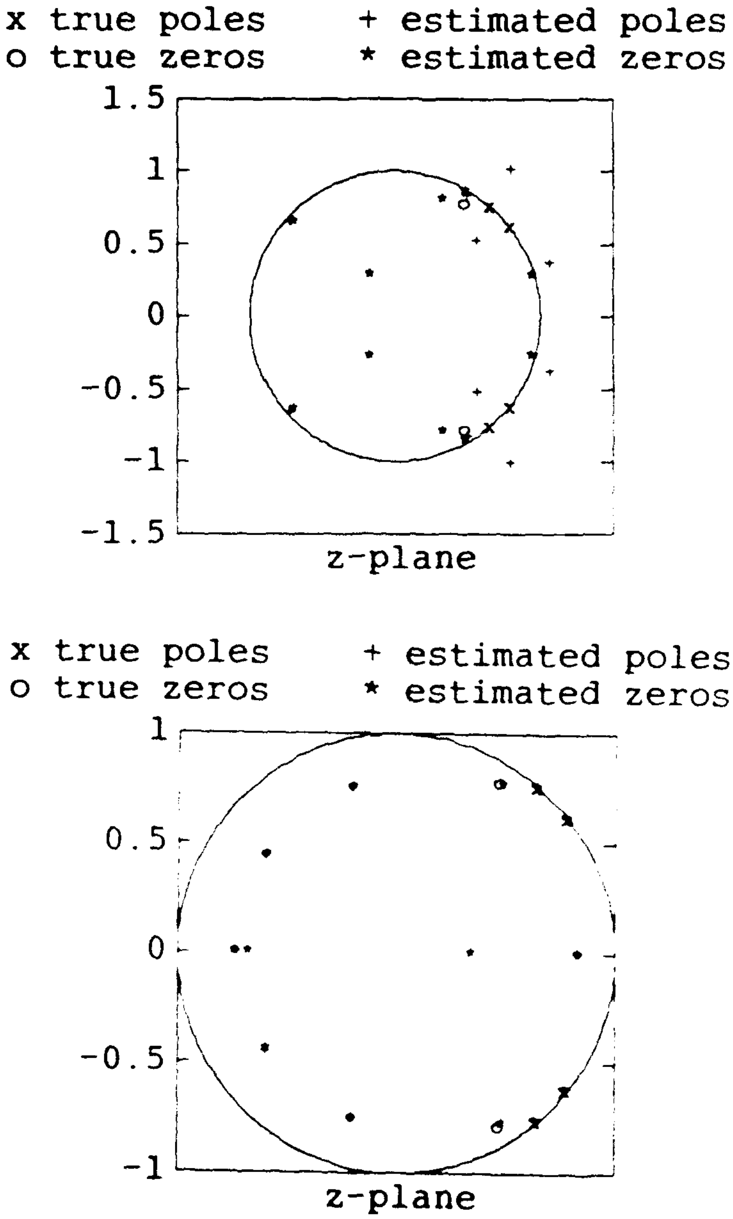
The above results and experience gained during extensive modeling trials lead to the following recommended strategy when modeling complex transients about which very little is known:

1. Be conservative in estimating the denominator order. Signals composed of narrowband components will generally be dominated by the few components highest in energy. Even if excess zeros are required to deal with a noisy signal (see the next section) a small number of excess zeros will usually suffice when using LSMYWE. Pole detection techniques such as those of Cadzow[Ref. 5], and Kumeresan, and Tufts [Ref. 21] may be helpful.

Method	Correctly Modeled? (Y/N)			
	P=10 Q=2	P=4 Q=10	P=10 Q=4	P=6 Q=10
Prony's Method	Y	Y	N	N
LSMYWE	Y	Y	N	N
Iterative Prefiltering	Y	Y	Y	Y
Corr Domain Iterative Pref	N	Y	Y	Y
Akaike MLE	Y	Y	Y	Y

TABLE 3.3: The effectiveness of thesis modeling methods on ARMA3 with varying overdetermination of model order. 'Y' indicates an exact time domain match was achieved and 'N' indicates a poor time domain match resulted.

2. Be expansive with estimates of numerator order. (Iterative prefiltering may be an exception to this. See Chapter IV.) Additional zeros can help compensate for mistakes made in data selection and effects of non-impulse excitation. Unneeded zeros will usually migrate away from the unit circle. Note that when using Prony's method or LSMYWE, one numerator order can be used when calculating the denominator coefficients and another numerator order can be chosen when finding the numerator coefficients. Using Shank's method with Prony's method or LSMYWE provides the most flexibility when a model is attempting to account for erroneous assumptions (e.g. data selection) which may have been made by the user. The above recomendations are primarily intended for Prony's method and LSMYWE. However, using any iterative technique requires reasonable initial estimates which are usually arrived at using linear estimation techniques.



(a)

(b)

Figure 3.9: Pole-zero plots for the sequence ARMA3 modeled with overdetermined model orders of P=10 and Q=10. (a) lsmywe and Durbin's method results in an unstable system, (b) iterative prefiltering provides an effective estimation of the correct poles and zeros and an excellent time domain match (not shown).

F. NOISE PERFORMANCE

Kay [Ref. 55] has shown that for an all-pole process, additive white noise has the effect of introducing zeros which migrate from the origin to the model poles as the signal-to-noise ratio is decreased. Alternatively, if zeros are not introduced into the model, poles move toward the origin as the signal-to-noise ratio is reduced [Ref. 56]. In either case the overall effect is a loss of spectral resolution. This result extends directly to pole-zero processes in white noise. Therefore one important feature of any modeling technique is it's ability to resolve spectral components in the presence of noise.

1. Rational Modeling of Noisy Data

Several authors have recently addressed the difficulties associated with modeling a time series in which additive white noise is present. Kay [Ref. 57] has noted that when the variance of the white noise can be estimated, its effect can be removed from the zeroth autocorrelation lag to improve the resolution of pole frequencies in correlation based autoregressive modeling. Alternatively, the zeroth autocorrelation lag may be avoided by using (2.18), sometimes called the *high order* Yule-Walker Equations, for $Q \geq P$ or by eliminating the rows containing the zeroth lag in certain least squares formulations [Ref. 44]. If the resulting high order Yule-Walker equations yield a matrix that is well conditioned, this procedure is equivalent to that of Kay above [Ref. 58]. Even with good correlation estimates, however, the matrix $\mathbf{R}_A$ is not guaranteed to be positive definite (i.e., invertible). Consequently, $\mathbf{R}_A$ is more likely to be a poorly conditioned matrix. This is not a problem in the least squares formulation, (2.19), since the very act of choosing such a formulation implies we expect a solution that is approximate.

A number of authors have noted that overdetermining the number of model poles can have a profound effect when modeling noisy signals [Ref. 55, 56, 21, 36, 39].

The explanation for this is that the extra poles are able to model the flat noise spectrum of the signal, preventing that portion of the signal from interfering with the poles needed for narrowband modeling. Finally, singular value decomposition has been used to aid resolution by overcoming the conditioning problems that can result from pole overdetermination and high order Yule-Walker equations [Ref. 5, 21, 36, 59, 60].

Little work has appeared which analyzes or demonstrates the effectiveness of iterative prefiltering in the presence of additive white noise. Stoica and Soderstrom [Ref. 61] have shown that iterative prefiltering will perform well for very long stochastic data sequences in the presence of additive white noise if reasonable initial estimates are available to start the iterations. They also show that for arbitrary initial estimates iterative prefiltering is not guaranteed to converge. Tufts and Kumeresan [Ref. 62] have demonstrated superior performance for approximate MLE over linear prediction methods when modeling sinusoids in white noise. Using the approximate MLE method of Box and Jenkins, Cadzow found that such a method performed comparably to the overdetermined high order Yule-Walker equations for the sum of two second order all-pole processes in white noise but with higher variance.

The above work suggests numerous possibilities in dealing with noisy impulse response data. The effectiveness of these techniques is evaluated empirically in the following section by modeling noisy test sequences.

2. Discussion and Examples

The examples used to illustrate modeling performance are summarized in Table 3.4. All noise modeling examples were conducted using the test sequence ARMA4 CL (see Table 3.1). Figure 3.10 shows that Prony's method has difficulty even when the SNR is as high as 30 dB although the excess poles introduced for Figure 3.10b dramatically increase modeling effectiveness. The moderate impact of

insuring that $Q \geq P$ is illustrated for LSMYWE in Figure 3.11 in that the higher frequency pole moves out toward the unit circle when the modeling order is altered from (4,2) to (4,4). As with Prony's method, overestimating the number of poles dramatically improves modeling in Figures 3.12 and 3.13. In Figure 3.14, the two previous noise modeling examples are again modeled with the smallest singular value set to zero to yield a data matrix of rank P. The technique is highly effective in both cases. The effectiveness of singular value decomposition when overdetermining the number of poles is illustrated by the examples in Figure 3.15. This is identical to the result described in [Ref. 21]. Discarding the excess singular values aids both in resolving the narrowband components present and in reducing the variance of the excess poles. Figures 3.16 and 3.17 show the hazards that are possible when using singular value decomposition. Any number of singular values discarded other than the those necessary to achieve a data matrix rank equal to the transfer function denominator order has undesirable side effects. Discarding too few singular values such as in the Figure 3.16 examples will resolve the desired narrowband components but also lead to excess poles near or even beyond the unit circle. This result will at best increase the likelihood of spurious narrowband components and can, in the worst case, lead to an unstable model. If too many singular values are discarded as in Figure 3.17b, the accuracy of narrowband component estimates will be badly degraded. Therefore, singular value decomposition, while potentially very useful in modeling transient signals in noise, must be used with caution when the true model order is unknown (which is basically always).

The iterative prefiltering, correlation domain iterative prefiltering, and Akaike MLE algorithms were initialized with several of the preceeding examples and the results are shown in and in Figures 3.18 and 3.19. Iterative prefiltering dramatically improved the narrowband modeling of each example with which it was initialized.

Figure 3.18a indicates that excess poles may cause problems by being close to the unit circle. However, the ability of iterative prefiltering to cancel excess zeros with excess poles demonstrated in this chapter may mitigate this effect. Correlation domain iterative prefiltering also improved the resolution of each noise example on which it was tried although its convergence was slower than and the bias of its results were higher than standard iterative prefiltering. Also, correlation domain iterative prefiltering showed a alarming tendency to place excess poles near the true poles on noisy signals. We experienced a great deal of difficulty in trying to model noisy signals with the Akaike MLE algorithm. The primary problem encountered was early algorithm termination due to an iteration which produced a non-minimum phase zero. Akaike MLE was able to improve the quality of a 'good' Prony estimate in Figure 3.19b. Finally, Figure 3.20 shows a pole-zero modeling result corresponding to the example in Figure 3.17a. Figure 3.20a shows the noisy signal used to during modeling and Figure 3.20b compares the resulting model to the original noiseless impulse response.

G. MODELING PERFORMANCE SUMMARY

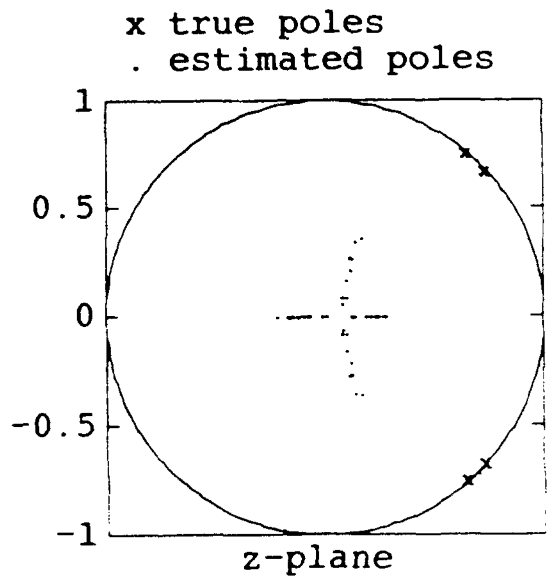
In modeling, the more that is known about the signal to be modeled, the better the choice of model structure and modeling algorithm that can be made. The focus of this thesis is the modeling of real world transient signals about which very little is known and whose characteristics can vary widely. The type of model is also set by the basic goal of this thesis, that is, a rational linear model. Under such circumstances, a sensible criterion in choosing a modeling algorithm is robustness. For numerator modeling, Shank's method is particularly robust in the sense that it provide the numerator coefficients which give the minimum norm, least squares fit based on the previously determined denominator coefficients. Shank's method is insensitive to time shifted data records and uncertainties in model order. Durbin's method is also

effective but suffers from the limitations that only minimum phase models are possible and the resulting models cannot account for time shifts. Since the Akaike MLE algorithm requires a minimum phase initial estimate, Durbin's method can be used provide an such an estimate. Equation (2.5) and spectral factorization are extremely sensitive to degraded signals (e.g. noisy signals) and so will not be considered further.

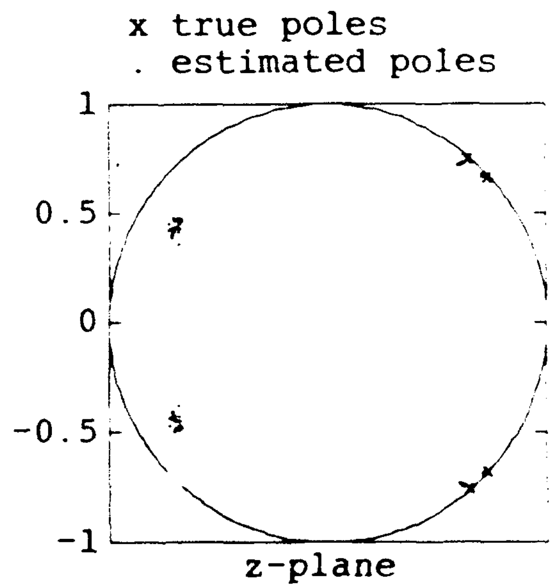
Denominator modeling with Prony's method is extremely sensitive to even small amounts of noise. This is only slightly less true of Akaike MLE. Also, Akaike MLE cannot continue to iterate if a non-minimum phase model estimate is encountered. In contrast, LSMYWE and iterative prefiltering are quite robust in noise and iterative prefiltering is particularly robust to model order overdetermination. Our conclusion is that for modeling the real world acoustic transients of the next chapter. LSMYWE with Shank's method and iterative prefiltering are likely to perform the best on the acoustic transients considered in the next chapter.

Method(order P,Q) (initialization)	SNR in dB	Bias (true minus model)	Variance of Pole Estimates	Figure
True Poles		$.6247 \pm .7509j$ $.7144 \pm .6711j$		
Prony (4,2)	30	$-.0598 \pm .0330j$ poor	$< 10^{-4}$ poor	3.10a
Prony (6,2)	30	$.0283 \pm .0186j$ $.0176 \mp .0074j$	$< 10^{-4}$ $< 10^{-4}$	3.10b
LSMYWE (4,2)	15	$.1788 \pm .1872j$ $.0346 \mp .0180j$	$< 10^{-4}$ $.0012 + .0014j$	3.11a
LSMYWE (4,4)	15	$.1475 \pm .1035j$ $.0386 \mp .0045j$	$< 10^{-4}$ $.0022 + .0019j$	3.11b
LSMYWE (6,2)	15	$.0053 \pm .0035j$ $.0027 \pm .0014j$	$< 10^{-4}$ $< 10^{-4}$	3.12a
LSMYWE (8,8)	10	$.0040 \pm .0054j$ $.0002 \pm .0037j$	$< 10^{-4}$ $< 10^{-4}$	3.12b
LSMYWE (8,8)	5	$.0316 \pm .0497j$ $.0147 \pm .0212j$	$.0002 + .0001j$ $.0008 + .0010j$	3.13a
LSMYWE (12,12)	5	$.0115 \pm .0077j$ $.0017 \pm .0063j$	$< 10^{-4}$ $< 10^{-4}$	3.13b
Prony (4,2) SVD	30	$-.0010 \pm .0040j$ $-.0016 \pm .0014j$	$.0001 + .0002j$ $< 10^{-4} + .003j$	3.14a
LSMYWE (4,4) SVD	15	$.0017 \mp .0005j$ $-.0007 \pm .0005j$	$< 10^{-4}$ $< 10^{-4}$	3.14b
Prony (8,2)	20	$-.0030 \pm .0387j$ $.0275 \pm .0188j$	$.0001 + .0001j$ $.0001 + .0001j$	3.15a
Prony (8,2) SVD	20	$-.0105 \pm .0043j$ $-.0059 \pm .0077j$	$< 10^{-4}$ $< 10^{-4}$	3.15b
LSMYWE (8,8) SVD	5	$-.0074 \pm .0100j$ $-.0070 \pm .0086j$	$.0006 + .0008j$ $.0003 + .0009j$	3.16a
LSMYWE (8,8) SVD	5	$-.0012 \pm .0069j$ $-.0062 \pm .0064j$	$.0004 + .0003j$ $.0004 + .0008j$	3.16b
LSMYWE (8,8) SVD	5	$-.0037 \pm .0024j$ $-.0049 \pm .0022j$	$.0001 + .0001j$ $.0002 + .0005j$	3.17a
LSMYWE (8,8) SVD	5	$-.0342 \pm .0526j$ $-.0507 \pm .0730j$	$.0001 + .0002j$ $< 10^{-4}$	3.17b
Iterative Prefiltering (4,4)	15	$-.0001 \pm .0001j$ $.0002 \pm .0001j$	$< 10^{-4}$ $< 10^{-4}$	3.18a
Iterative Prefiltering (8,8)	5	$-.0001 \pm .0011j$ $-.0006 \pm .0002j$	$< 10^{-4}$ $< 10^{-4}$	3.18b
Corr Domain (8,8) Iter Pref	5	$-.0054 \pm .0030j$ $.0002 \pm .0017j$	$< 10^{-4}$ $< 10^{-4}$	3.19a
Akaike MLE (6,2)	30	$-.0008 \mp .0009j$ $-.0004 \mp .0010j$	$< 10^{-4}$ $< 10^{-4}$	3.19b

TABLE 3.4: Examples of pole-zero modeling performance of the impulse response test sequence ARMA4 CL in added noise.



(a)



(b)

Figure 3.10: Model pole scatter plots of twenty trials illustrating the effectiveness of overdetermining the number of model poles using Prony's method. Test sequence is ARMA4 CL with 30 dB of noise. (a) Using the correct model order, (4,2) and (b) Using two excess poles, model order (6,2).

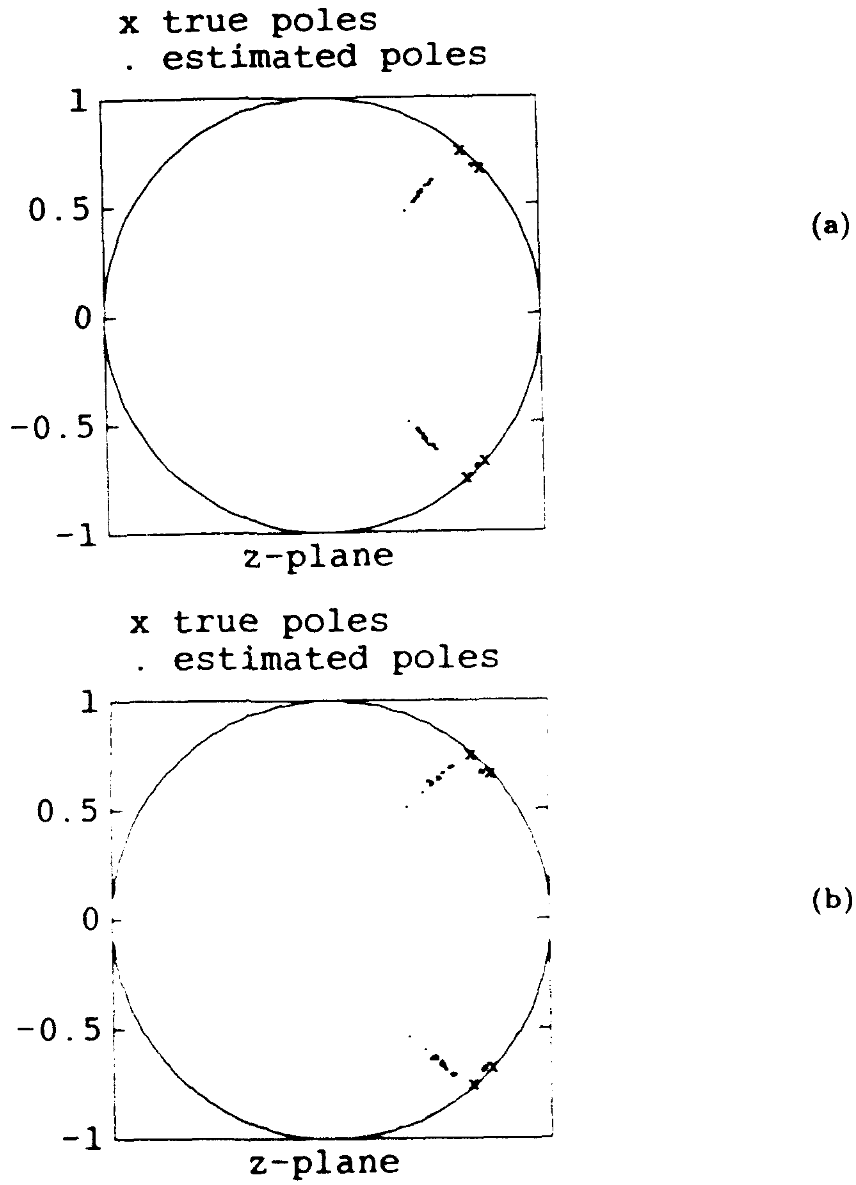
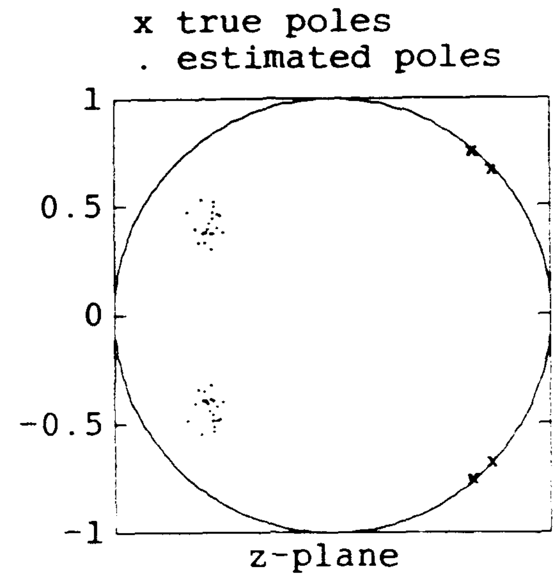
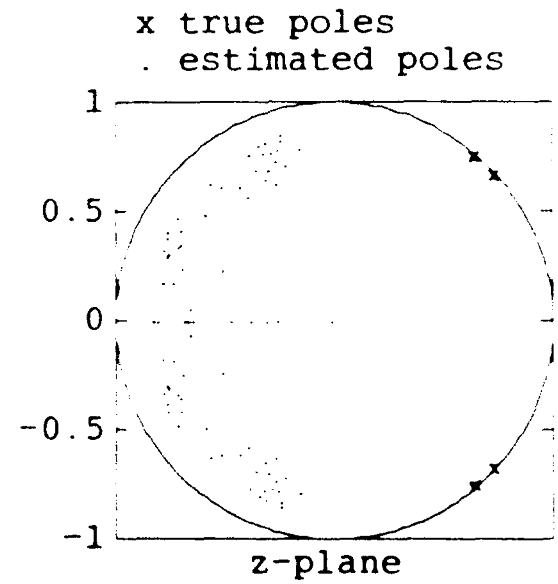


Figure 3.11: Model pole scatter plots of twenty trials illustrating the moderate benefits of zeroth lag correlation compensation by choosing $Q = P$ when using LSMYWE. Test sequence is ARMA4 CL with 30 dB of noise. (a) Using the correct model order, (4,2) and (b) model order (4,4).

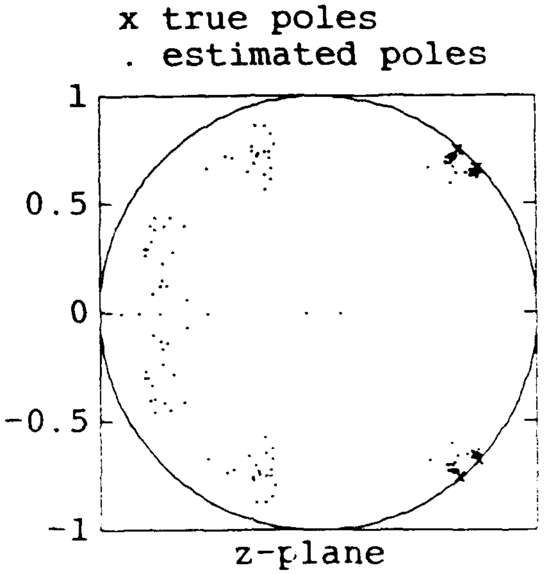


(a)

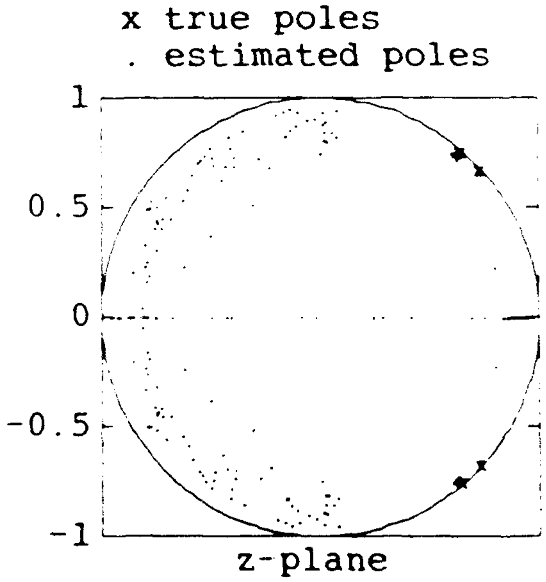


(b)

Figure 3.12: Model pole scatter plots of twenty trials illustrating the dramatic benefits of using excess poles with LSMYWE. Test sequence is ARMA4c. (a) Model order (6,2) with 15 dB of added noise and (b) model order (8,8) with 10dB of added noise.

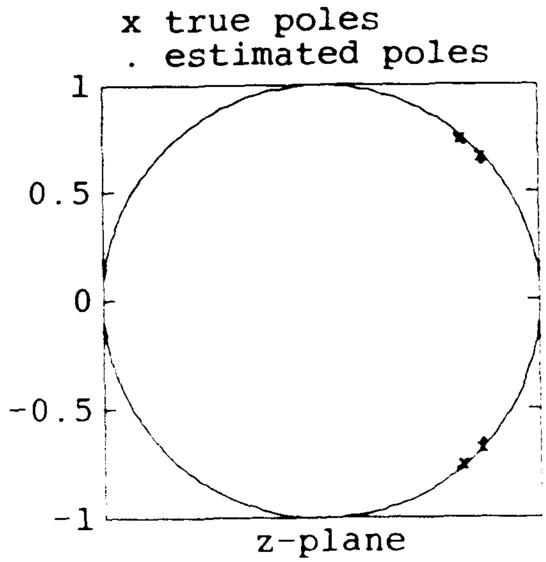


(a)

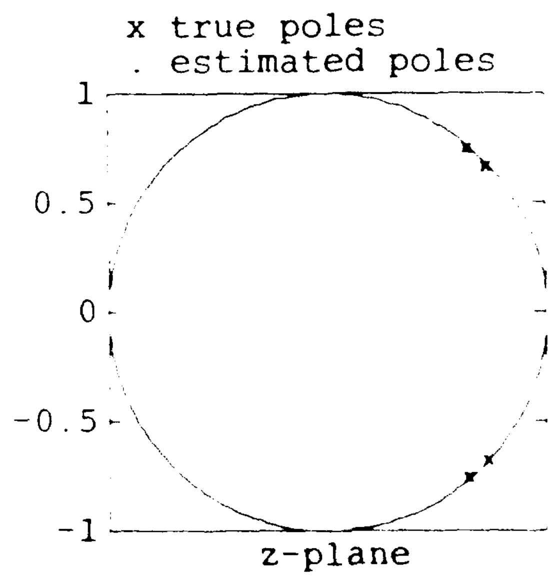


(b)

Figure 3.13: Model pole scatter plots of twenty trials illustrating the dramatic benefits of using excess poles with LSMYWE. Test sequence is ARMA4 CL. (a) Model order (8,8) with 5 dB of added noise and (b) model order (12,12) with 5 dB of added noise.



(a)



(b)

Figure 3.14: Model pole scatter plots of twenty trials illustrating the dramatic benefits of setting the smallest singular value of the data matrix to zero (i.e. adjust data matrix rank to P) for Prony's method and LSMYWE. Test sequence is ARMA4 CL. (a) Prony's method for model order (4,2) with 30 dB of added noise and (b) LSMYWE for model order (4,4) with 15 dB of added noise.

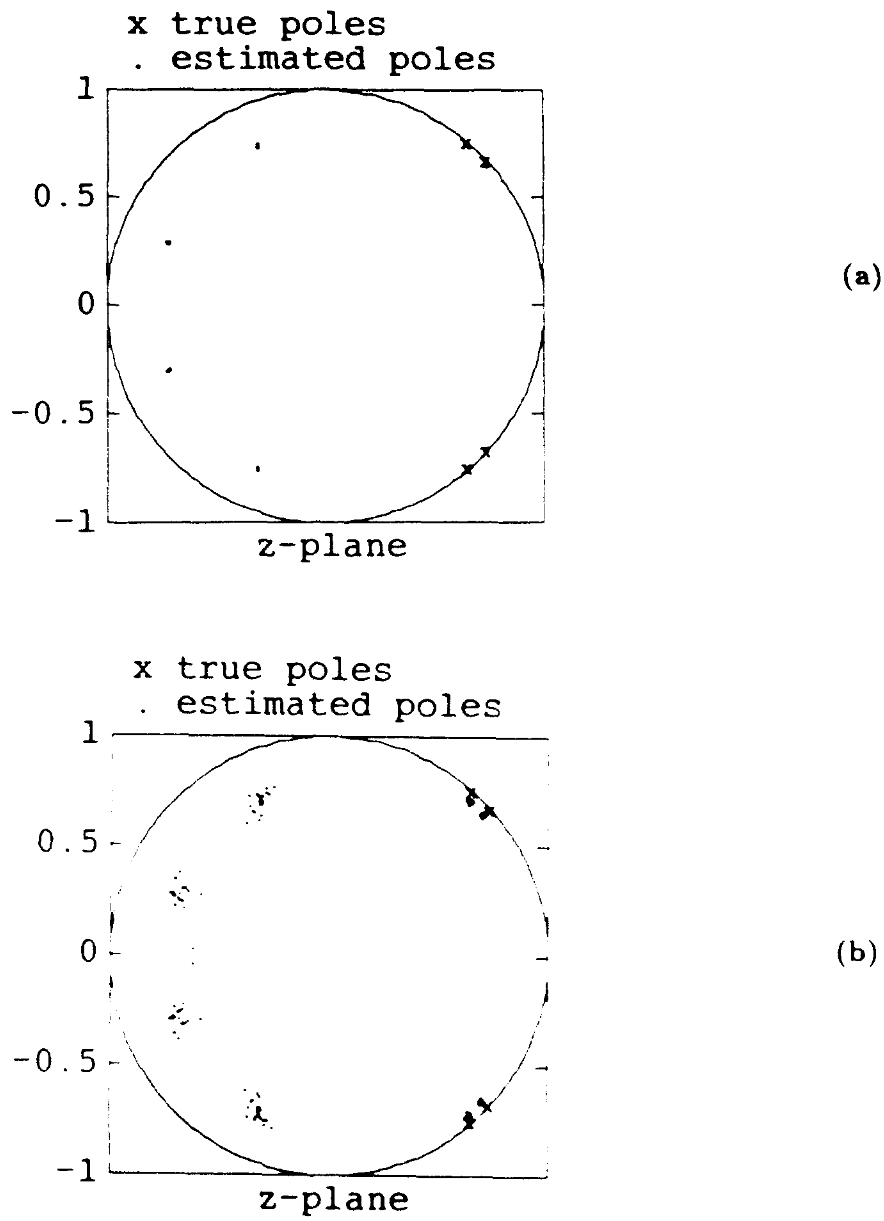
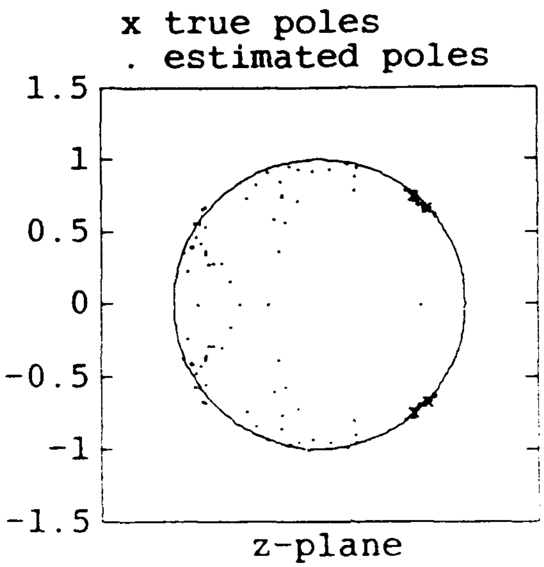
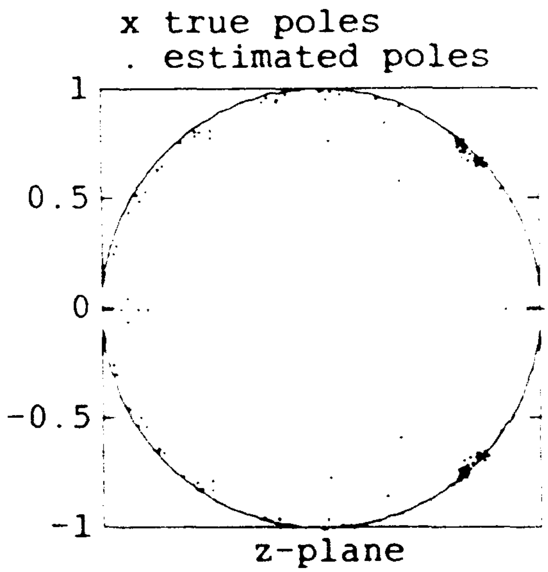


Figure 3.15: Model pole scatter plots of twenty trials illustrating the effect of adjusting the data matrix rank when using excess poles for Prony's method. Test sequence is ARMA4c. (a) Model order (8,2) with 20 dB of added noise and no rank adjustment and (b) model order (8,2) with 20 dB of added noise with rank adjusted to $P_{true} = 4$ using singular value decomposition.

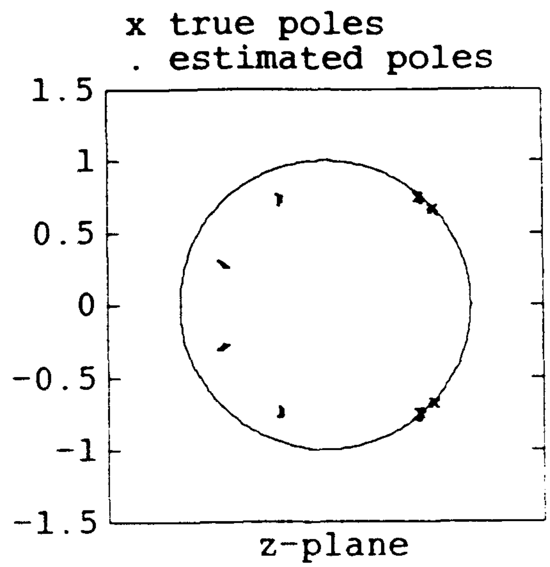


(a)

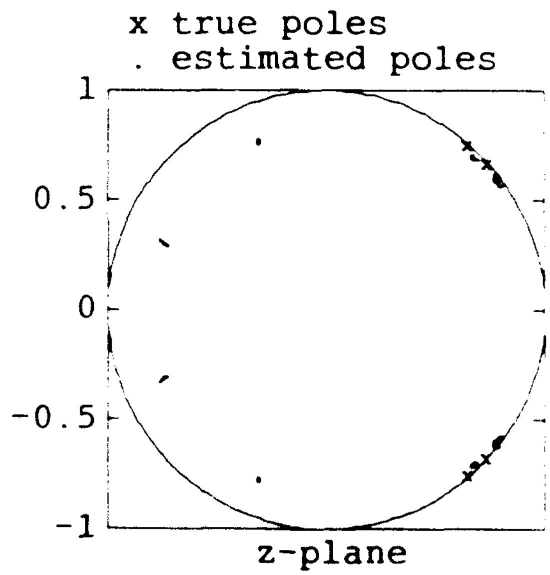


(b)

Figure 3.16: Model pole scatter plots of twenty trials illustrating the effect of adjusting the data matrix rank when using excess poles for LSMYWE. Test sequence is ARMA4 CL. (a) Model order (8,8) with 5 dB of added noise with rank adjusted to $P_{true} + 4 = 8$ and (b) model order (8,8) with 5 dB of added noise with rank adjusted to $P_{true} + 2 = 6$.

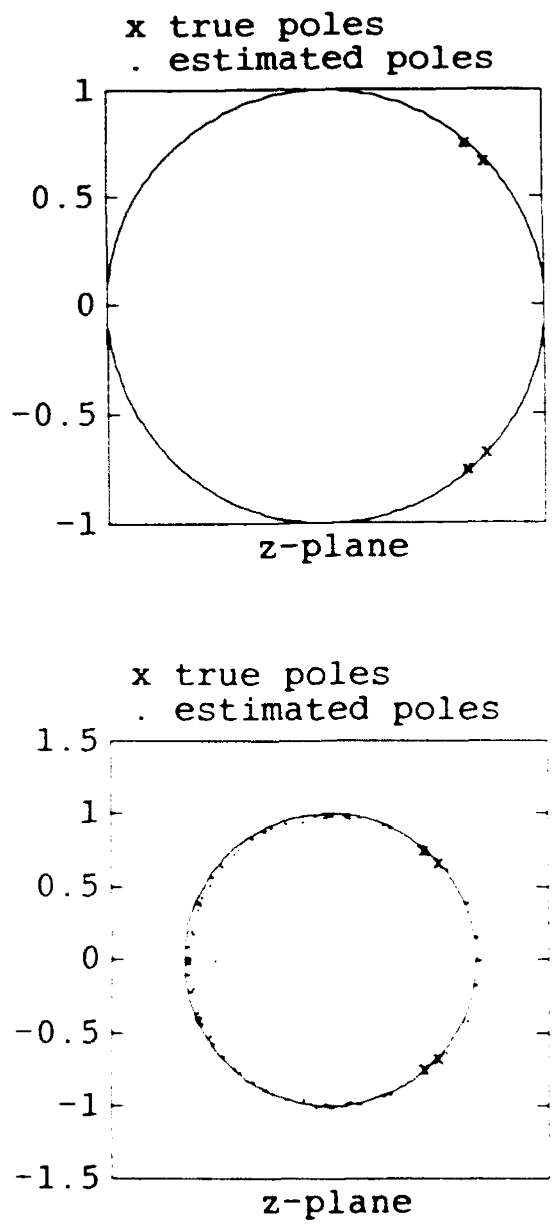


(a)



(b)

Figure 3.17: Model pole scatter plots of twenty trials illustrating the effect of adjusting the data matrix rank when using excess poles for LSMYWE. Test sequence is ARMA4 CL. (a) Model order (8,8) with 5 dB of added noise with rank adjusted to $P_{true} = 4$ and (b) model order (8,8) with 5 dB of added noise with rank adjusted to $P_{true} - 1 = 3$.



(a)

(b)

Figure 3.18: Model pole scatter plots of twenty trials illustrating the ability of iterative prefiltering to improve the resolution of an LSMYWE estimate. In both cases the iterative prefiltering algorithm was initialized using LSMYWE and Durbin's method. Test sequence is ARMA4 CL. (a) Model order (4,4) with 15 dB of added noise and (b) model order (8,8) with 5 dB of added noise.

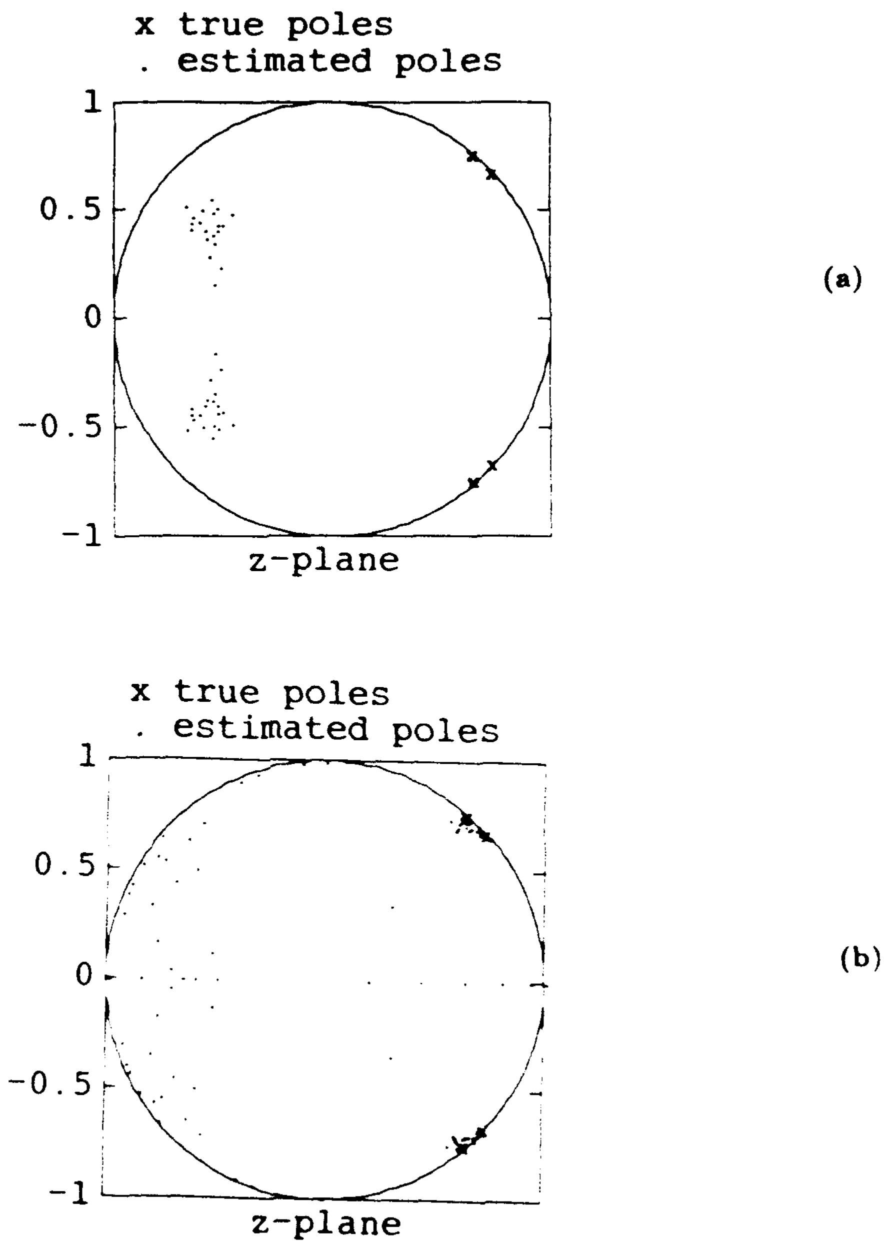


Figure 3.19: Model pole scatter plots of twenty trials on test sequence ARMA4 CL illustrating (a) the ability of correlation domain prefiltering to improve the resolution of an LSMYWE initial estimate, order (8,8), 5 dB added noise and (b) the ability of Akaike MLE to improve the resolution of a Prony's method initial estimate, model order (6,2), 30 dB added noise.

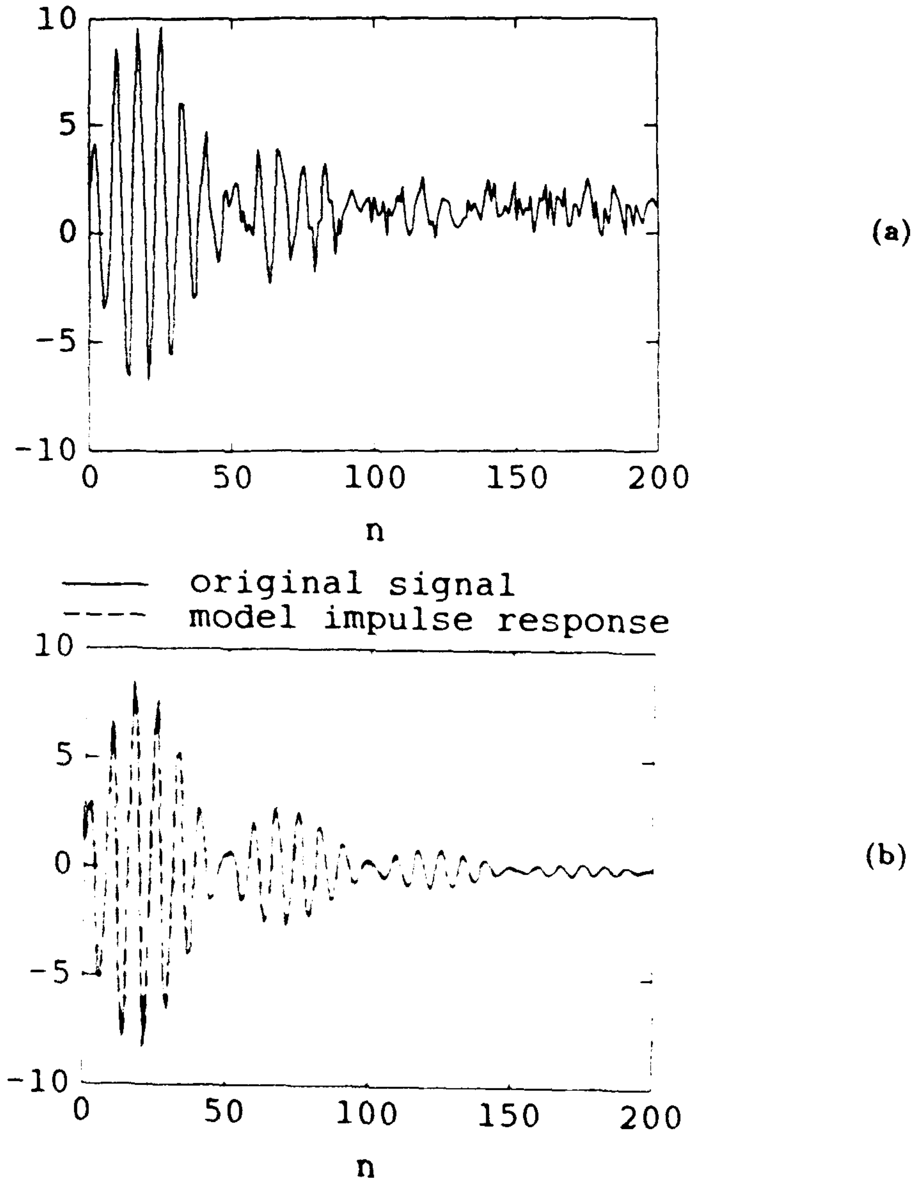


Figure 3.20: Pole-zero model of the ARMA4 CL test sequence with 5 dB of additive white noise using LSMYWE and the smallest singular value discarded. The MA part was found using least squares identification. This example corresponds to the example in Figure (3.15a). (a) The noisy sequence and (b) the estimated model impulse response and the original noiseless sequence. Model order (8,8).

IV. LINEAR MODELING OF ACOUSTIC TRANSIENTS

A. ACOUSTIC DATA—GENERAL

The laboratory generated acoustic transient data available for this study were generated in six different ways using ordinary laboratory objects. The data records and their method of generation are summarized in Table 4.1. Each transient will be refered to by the object used or the action performed to generate that particular transient such as 'book' or 'slam'. The six data records modeled in this section are shown in Figures 4.1a-f. The range of data that was actually modeled is listed in Table 4.1. The sampling rate for the data is unknown but is not required since there is no need to infer the specific characterics of the acoustic source or the acoustic enviroment.

Data Name	Generation Technique	Indices of Modeled Data (begin:end)
Book	Dropped Book	(55:454)
Slam	Slammed Book	(15:414)
Plate	Struck Metal Plate	(5:704)
Ruler	Book Struck with Ruler	(501:650)
Wood	Clapped Wooden Blocks	(171:320)
Wrench	Dropped Wrench	(101:250)

TABLE 4.1: Summary of Acoustic Transient Data and method of generation.

B. ACOUSTIC TRANSIENT MODELING RESULTS

Iterative prefiltering and correlation domain iterative prefiltering were found to yield the most effective time domain match (in terms of squared error) between the

original signal and the impulse response of the estimated model. LSMYWE with the smallest singular value set to zero, as in Figures 3.13a,b, was used to find the initializing model poles for the iterative algorithms. Removing the smallest singular value allowed the LSMYWE algorithm to place zeros much closer to the unit circle than was possible when all singular values were used. Prony's method and the Akaike MLE algorithm were also used to model laboratory transients. However, their performance was generally poor and they will not appear in the remainder of this chapter. As noted in Chapter III, the problem with Prony's method is that it cannot easily model highly resonant frequencies, that is, zeros close to the unit circle, in the presence of even small amounts of noise unless a large number of excess poles are used and numerous singular values are discarded. This procedure is burdensome when initializing iterative algorithms which do not require these excess poles. The primary difficulty with Akaike MLE is its inability to handle zeros outside the unit circle. The initializing model zeros were found using Shank's method which was by far the most robust algorithm for finding numerator coefficients of the methods tested in Chapter Three. In addition to the time domain plots of model impulse response, a pole-zero model spectrum and pole-zero plot is be provided for each transient so that their differing characteristics can be observed. All spectra were generated by squaring the FFT magnitude of either the model coefficients or the signal being modeled. Model order was chosen based on the best educated guess of the author in accordance with the recommendations in Chapter III, and augmented by trial and error.

The LSMYWE model used to initialize the iterative prefiltering algorithm for the Slam transient is shown in Figures 4.2a and 4.3a. The corresponding iterative prefiltering model shown in Figures 4.2b and 4.3b. Figure 4.4a shows the iterative prefiltering model spectrum. Figure 4.4b illustrates a characteristic of the iterative prefiltering algorithm that was observed throughout the modeling of acoustic

transients. Namely, when an excess number of model parameters are used, the error between the model impulse response and the signal being modeled increases dramatically, mainly at the beginning of the signal. The subsequent application of Shank's method is effective in reducing this initial error. This effect is shown in Figure 4.5a. In general, the application of Shank's method as the last step in the modeling process was found to reduce the the mean squared error between the original signal and the model impulse response to some degree for all modeling methods. The sensitivity to excess parameters shown by iterative prefiltering does not affect correlation domain iterative prefiltering. Note that excess model zeros cause no difficulties in Figure 4.5b. For the Book transient, two modeling trials are shown. Figures 4.6 and 4.7 show the best time domain match obtained using iterative prefiltering and also an LSMYWE, Shank's method, respectively. Although the LSMYWE model does not achieve as effective an impulse response match as iterative prefiltering, with SVD it is more sensitive to the low energy, high frequency component present in the Book transient at approximately $\frac{\pi}{2}$. The best model of Ruler is shown in Figure 4.8. The model spectra for Book and Ruler appear in Figure 4.9.

The assumption that the signal being modeled is a system impulse response is problematic for the Plate, Wood, and Wrench signals since they do not exhibit the rapid decay usually associated with an impulse response. However, it is still possible to achieve a resonable time domain match over small segments of each signal. This result is illustrated in the models of in Figures 4.10, 4.11, 4.13, and 4.14. The model spectra for Wood and Wrench are shown in Figure 4.15. The reason that correlation domain iterative prefiltering was used for the Plate, Wood, and Wrench signals instead of standard iterative prefiltering can be illustrated by comparing the two techniques on a segment of the Plate signal. Although the impulse response error of the two models in Figures 4.10 and 4.11 are nearly identical, Figure 4.12 shows

that correlation domain iterative prefiltering clearly outperformed standard iterative prefiltering in reproducing the spectrum of the original sequence. In fact, at the more suitable model order of (16,16), iterative prefiltering would not converge but instead oscillated in a region of convergence for any model order over (12,12). Figure 4.16b shows the model impulse response obtained when the large segment of Plate shown in Figure 4.16a is modeled using iterative prefiltering. Although the time domain and spectral properties of the model relative to the original signal are considerable poorer than those obtained for a short segment, the model does clearly share many of the features of the original signal.

C. ACOUSTIC SIGNAL MODELING SUMMARY

The modeling results obtained in the previous section of this Chapter indicate the possible utility of pole-zero modeling algorithms with regard to modeling transient signals. Signals with decaying narrowband components (e.g. Slam, Book, and Ruler) and signals with substained narrowband components (e.g. Plate, Wood, and Wrench) can be modeled as the impulse response of a rational linear system. Robust modeling algorithms are available which can effectively deal with the many uncertainties associated with real world signals. Although the goal of achieving an exact time domain match between the original signal and the pole-zero model impulse response was not realized for any of the acoustic signals in this chapter, in all cases the degree of match obtained clearly indicates that many signal characteristics are described by the model.

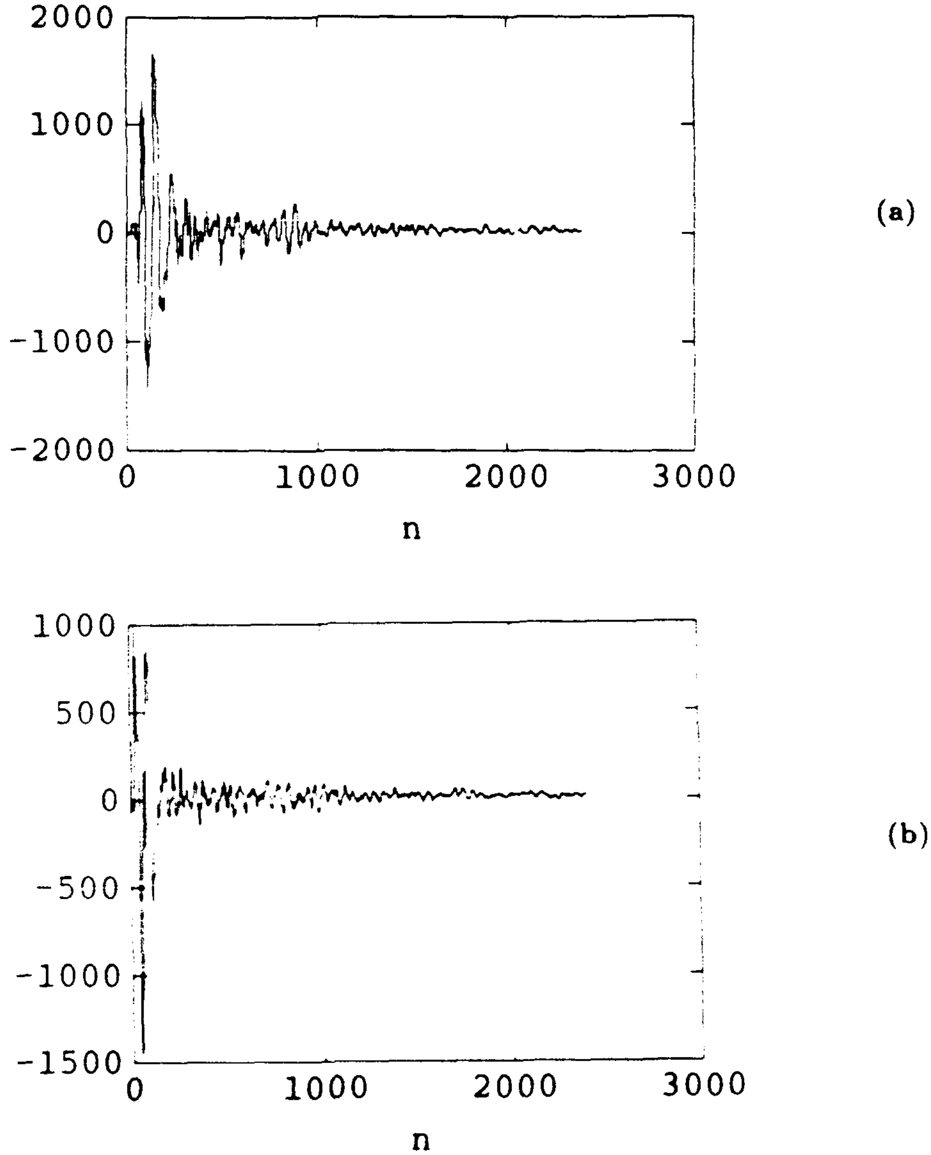


Figure 4.1: Laboratory generated acoustic transient data: (a) Slam and (b) Book.

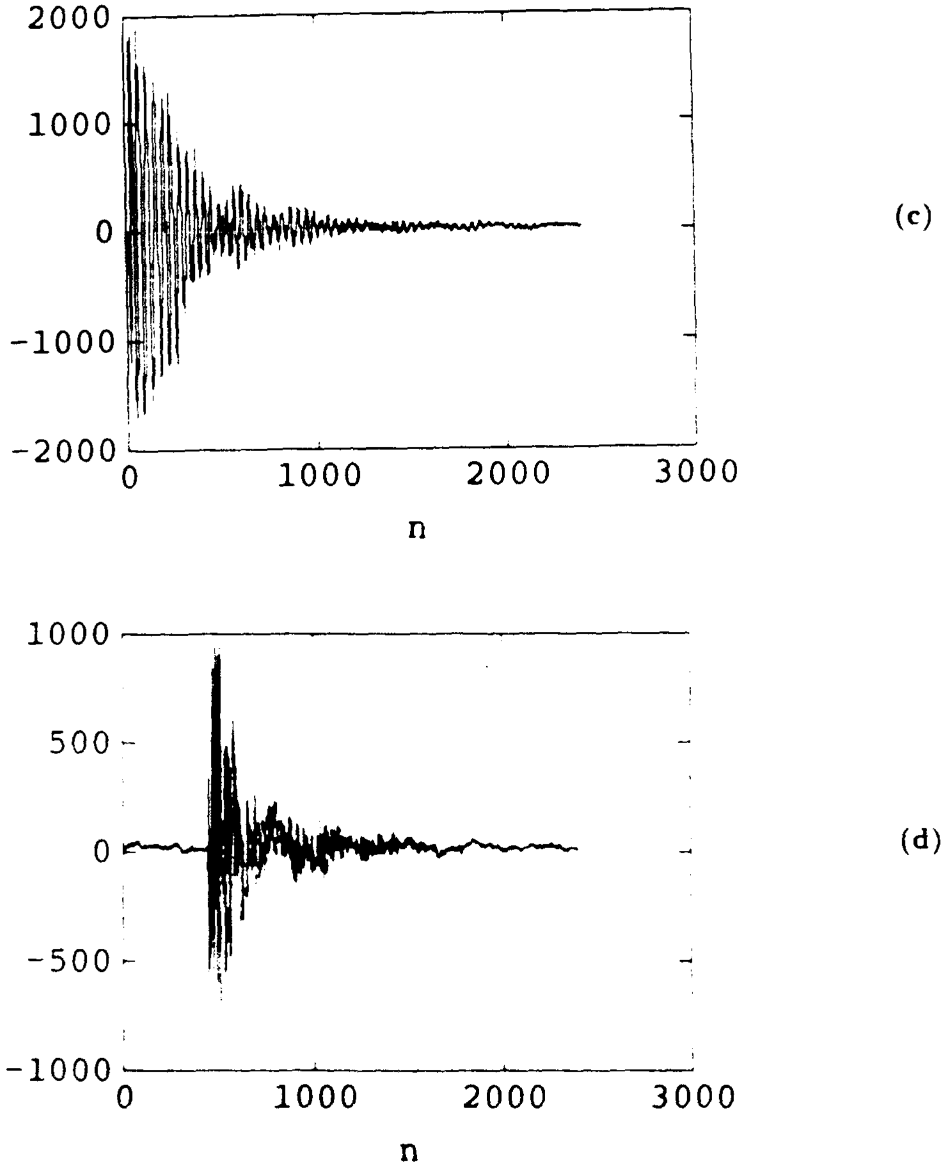
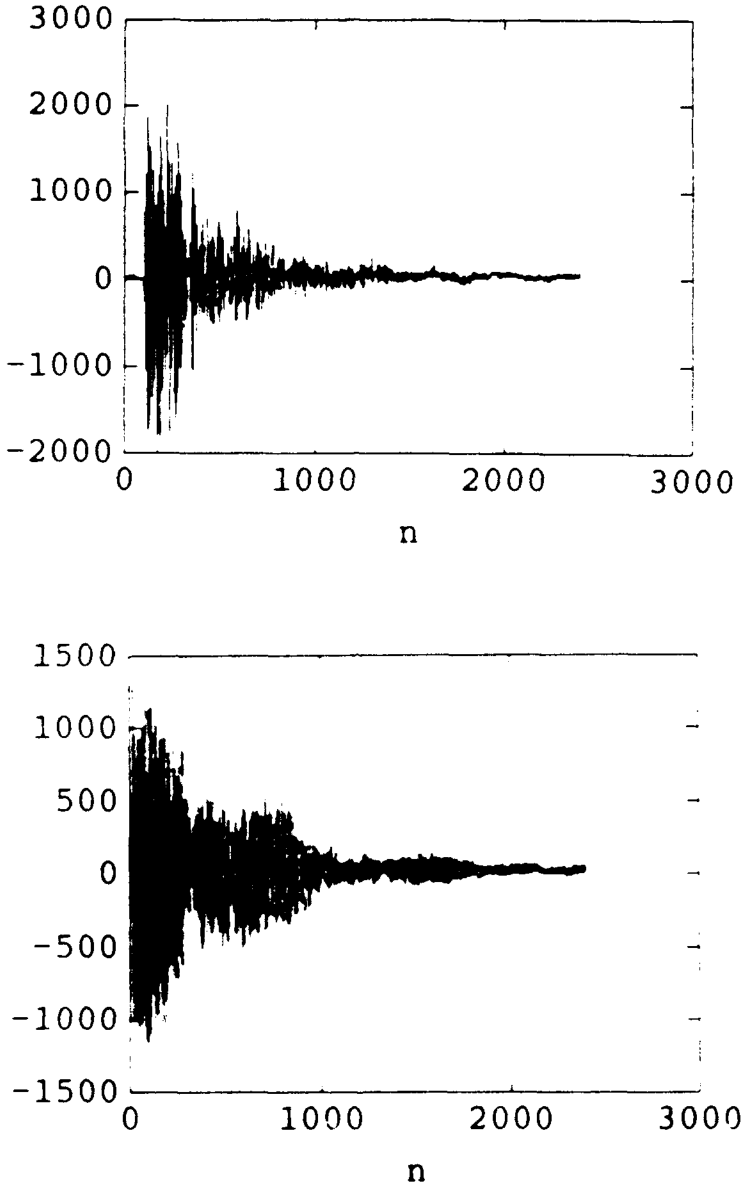


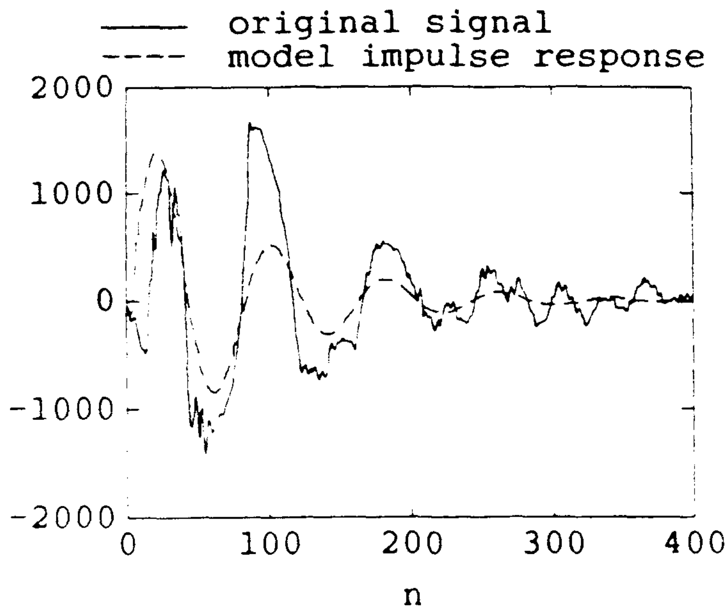
Figure 4.1: continued Laboratory generated acoustic transient data: (c) Ruler and (d) Plate.



(e)

(f)

Figure 4.1: continued Laboratory generated acoustic transient data: (e) Wood and (f) Wrench.



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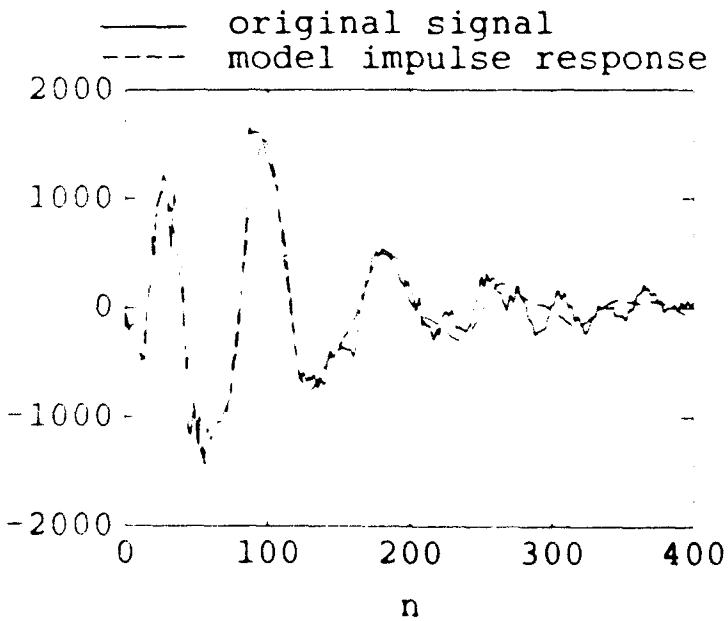
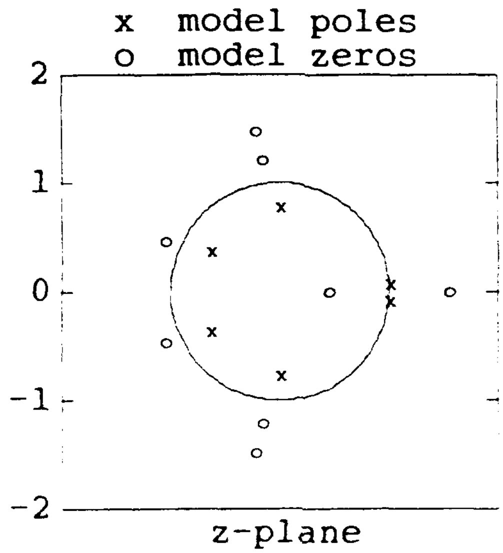


(b)

Figure 4.2: Modeling the transient Slam—model impulse response versus the original signal. Model order (6,8). (a) LSMYWE and Shank's method and (b) iterative prefiltering initialized with (a).



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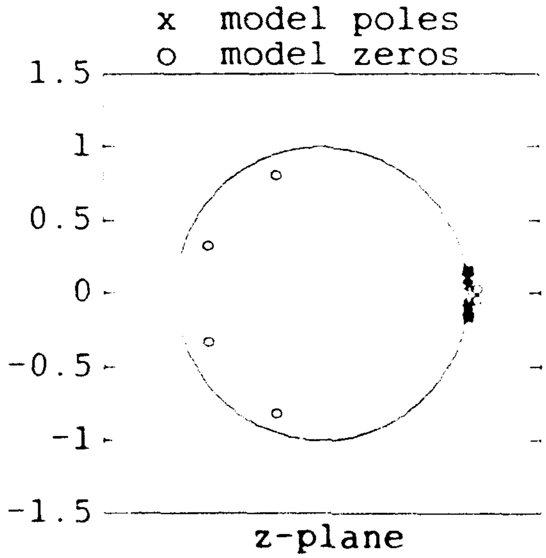


(b)

Figure 4.3: Modeling the transient Slam—model pole-zero plots. Model order (6,8). (a) LSMYWE with SVD and Shank's method and (b) iterative prefiltering initialized with (a).

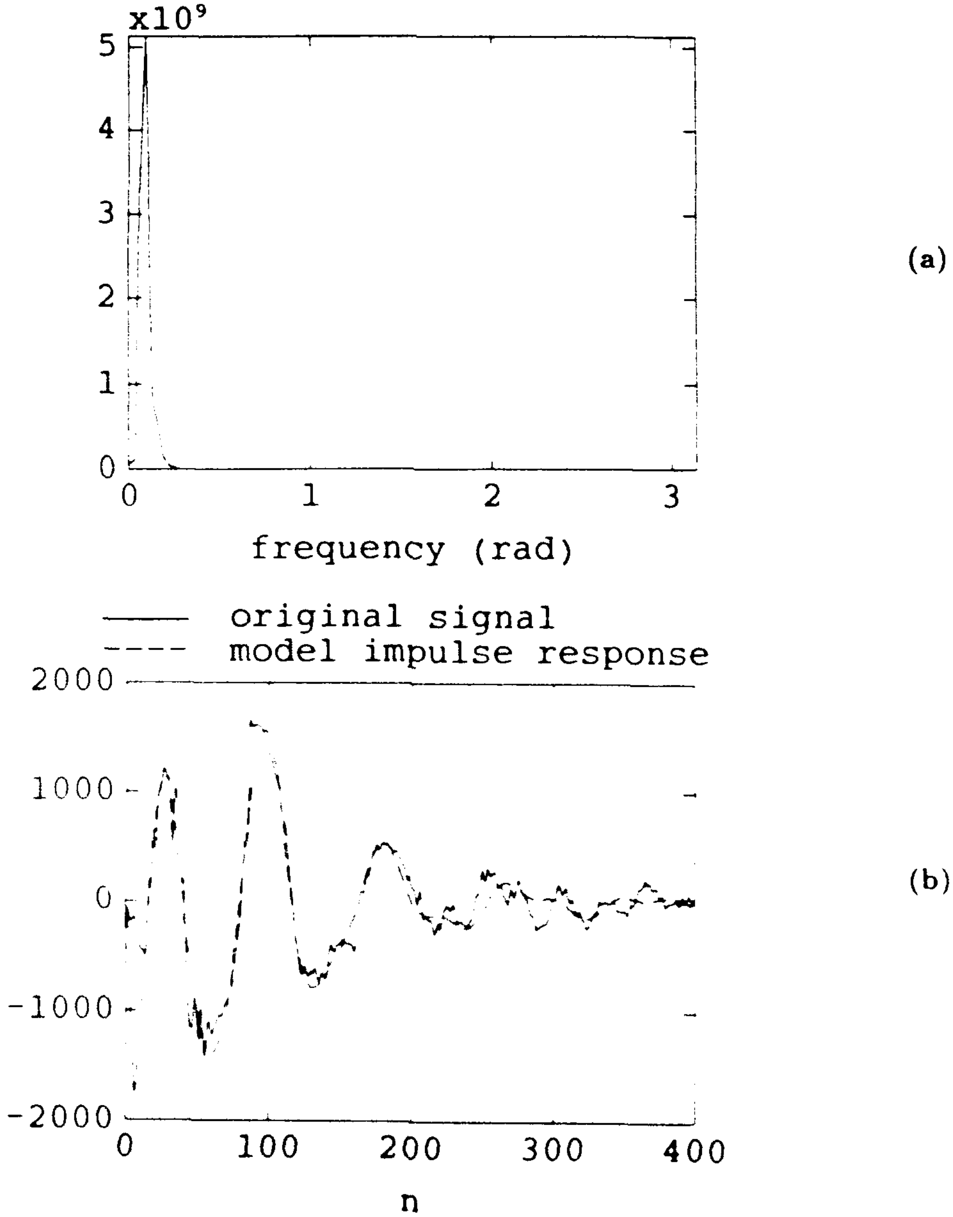
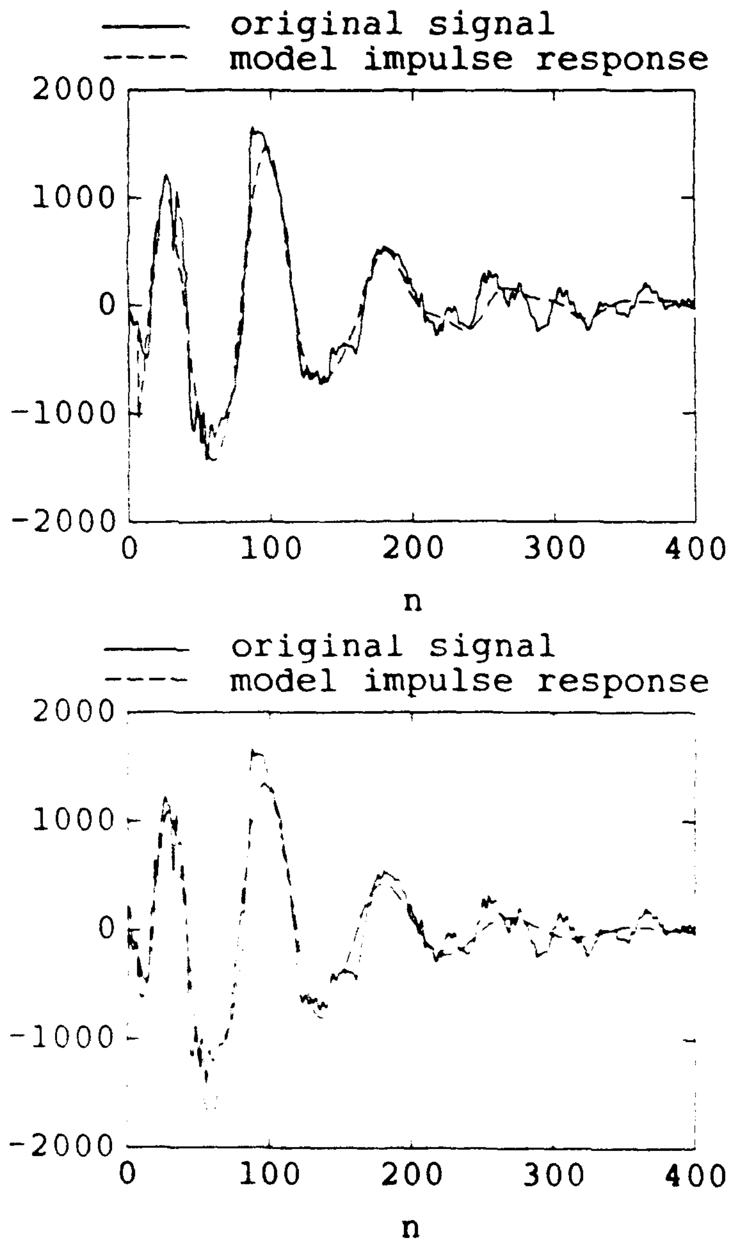


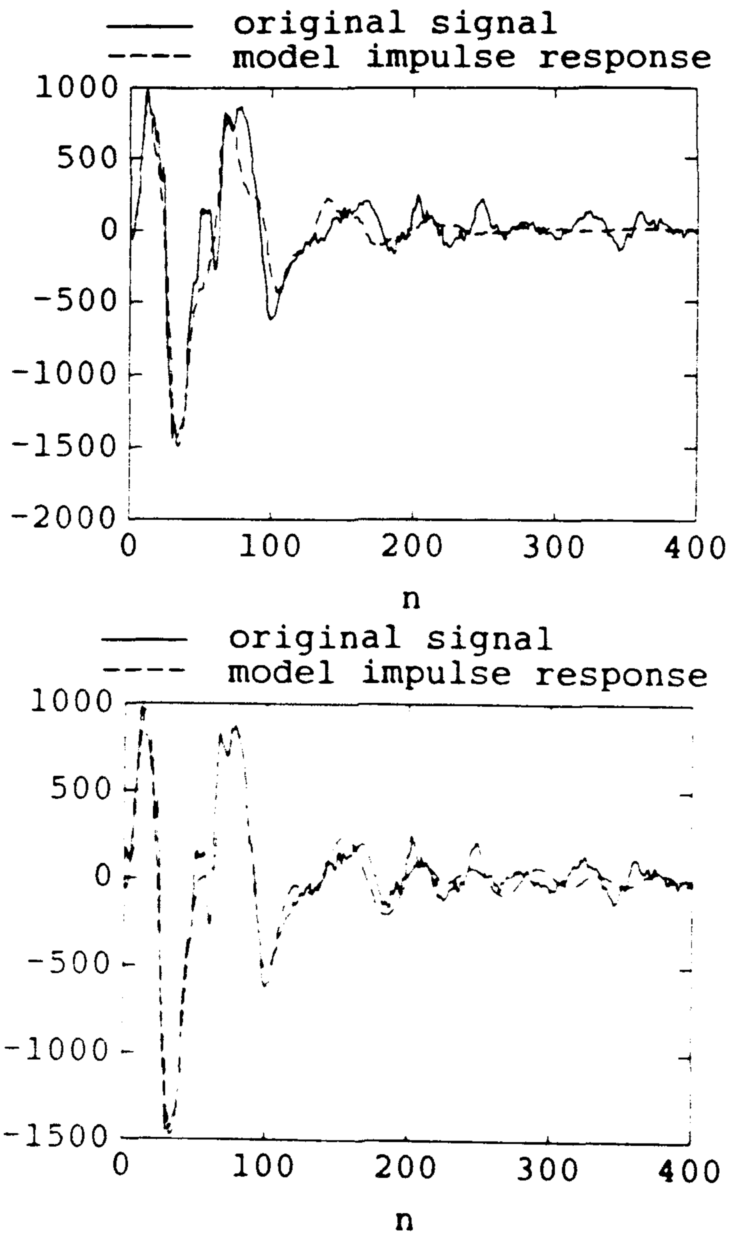
Figure 4.4: Modeling the transient Slam—model spectrum and and over-parametrized iterative prefiltering model impulse response versus the original signal. (a) Model spectrum for iterative prefiltering with model order (6,8) and (b) iterative prefiltering for model order(6,12). Note how excess model parameters cause error at the beginning of the iterative prefiltering model.



(a)

(b)

Figure 4.5: Modeling the transient Slam—model impulse response versus the original signal, overparameterized modeling effects. (a) The application of Shanks method to the iterative prefiltering model of order (6,12) reduces the error at the beginning of the model impulse response and (b) correlation domain iterative prefiltering for model order (6,12) can effectively use the excess model parameters.



(a)

(b)

Figure 4.6: Modeling the transient Book—model impulse response versus the original signal. Model order (6,6). (a) LSMYWE with SVD and Shank's method and (b) iterative prefiltering initialized with (a). Note the sensitivity of LSMYWE with SVD to the high frequency components present.

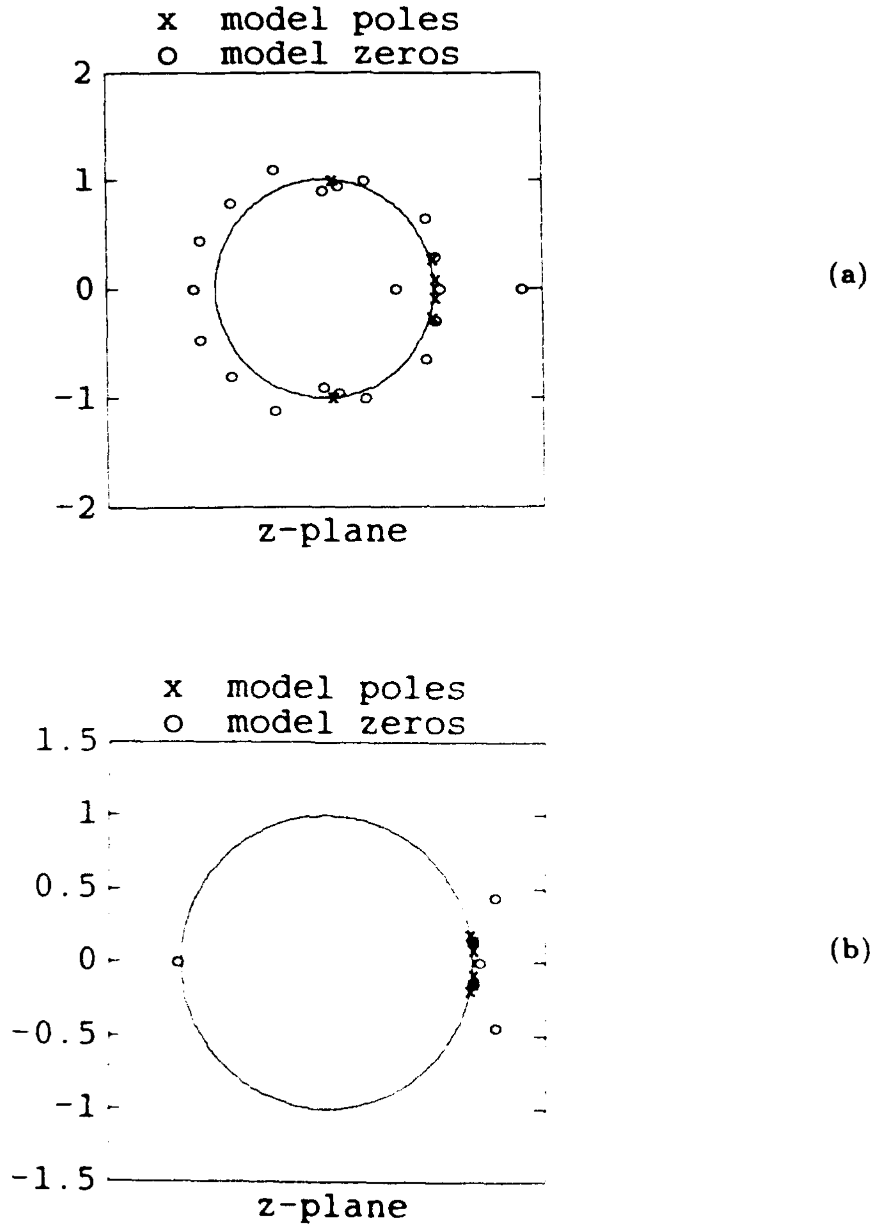
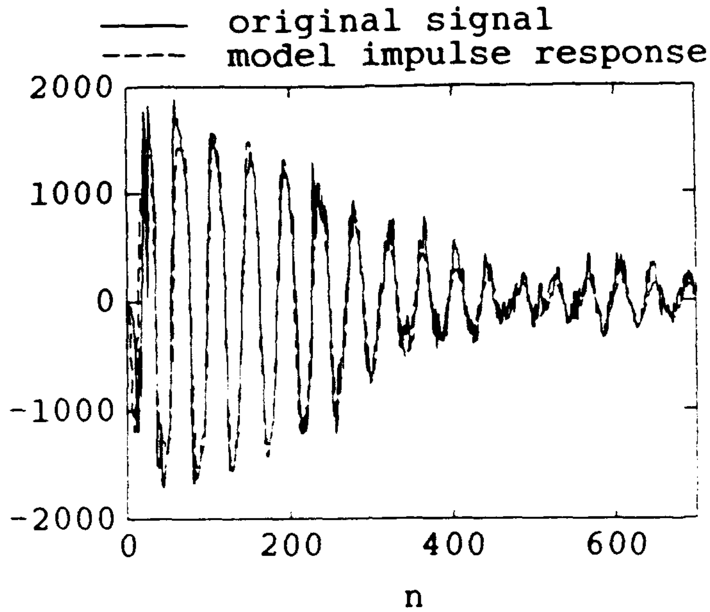


Figure 4.7: Modeling the transient Book—model pole-zero plots. Model order (6,6). (a) LSMYWE with SVD and Shank's method and (b) iterative prefiltering initialized with (a). Note the sensitivity of LSMYWE with SVD to the high frequency pole pair.



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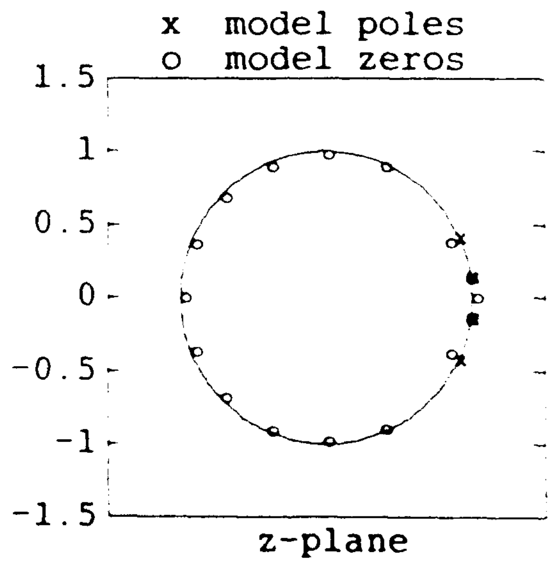


(b)

Figure 4.8: Modeling the transient Ruler—Model order (6,16). (a) Iterative prefiltering model impulse response versus the original signal and (b) the corresponding model pole-zero plot. Note that the two beating sinusoids have been effectively modeled.

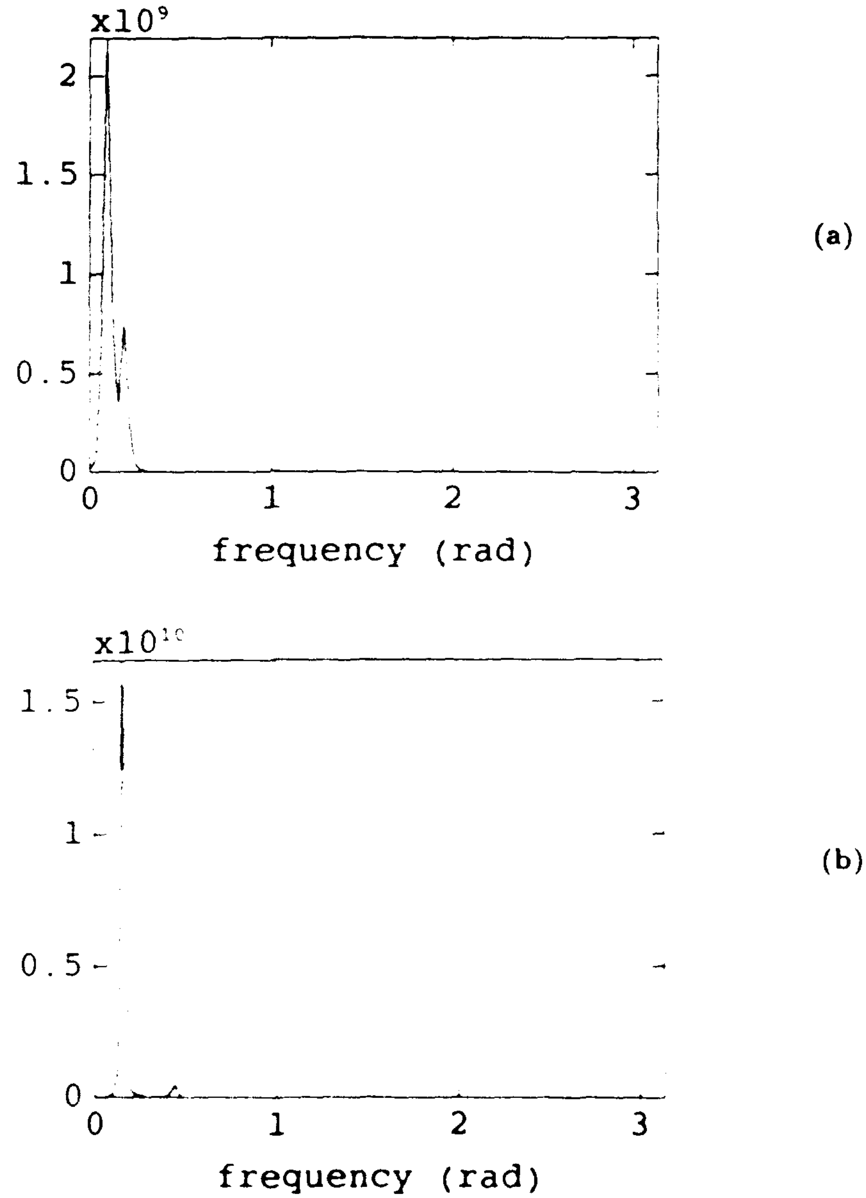
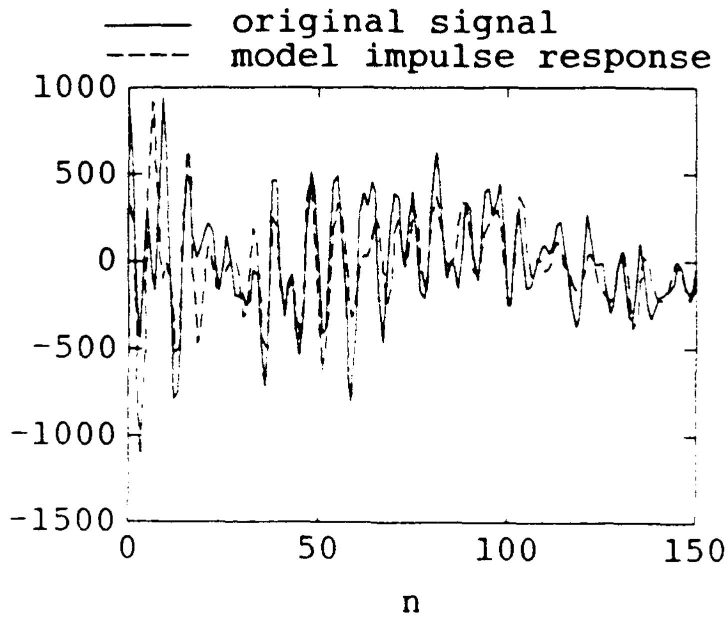


Figure 4.9: Modeling the transient Book and Ruler—model spectrums. (a) The iterative prefiltering model of order (6,6) for the transient Book and (b) the iterative prefiltering model of order(6,12) for the transient Ruler.



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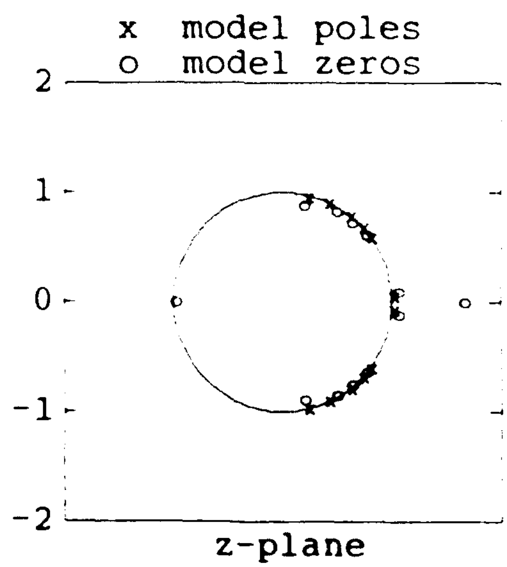


(b)

Figure 4.10: Modeling the transient Plate—Model order (12,12). (a) Iterative prefiltering model impulse response versus the original signal and (b) the corresponding model pole-zero plot.

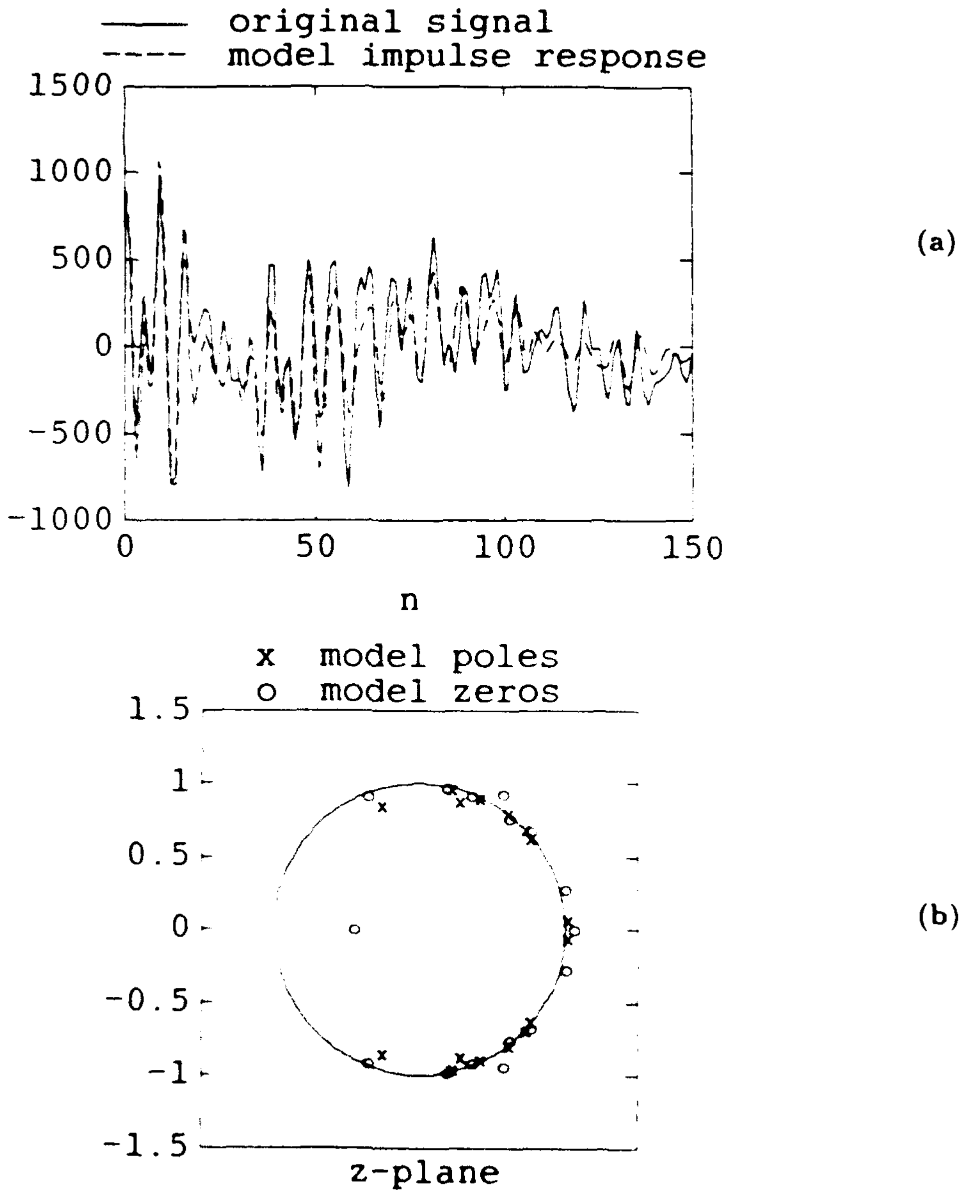


Figure 4.11: Modeling the transient Plate—Model order (16,16). (a) Correlation domain iterative prefiltering model impulse response versus the original signal and (b) the corresponding model pole-zero plot.

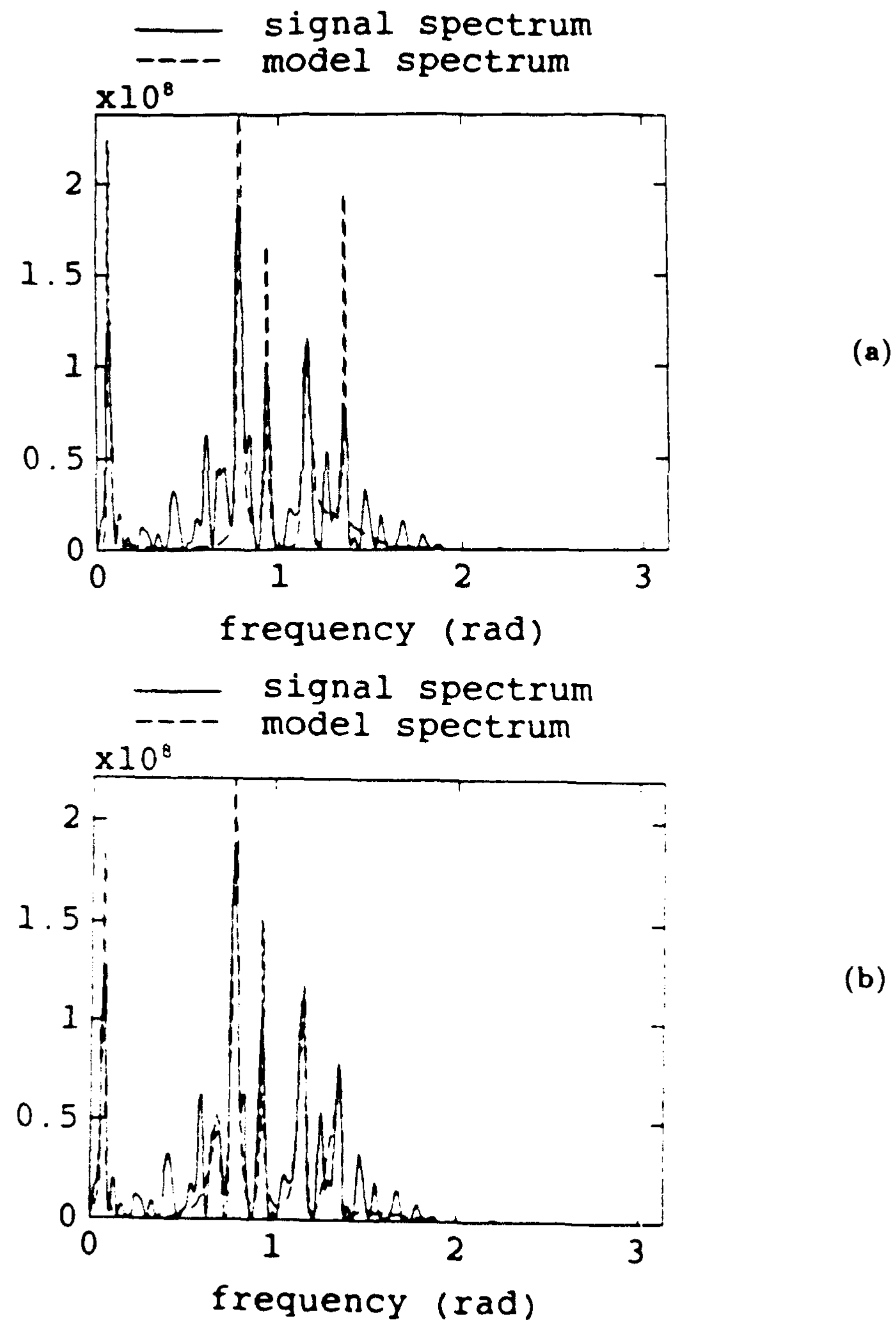
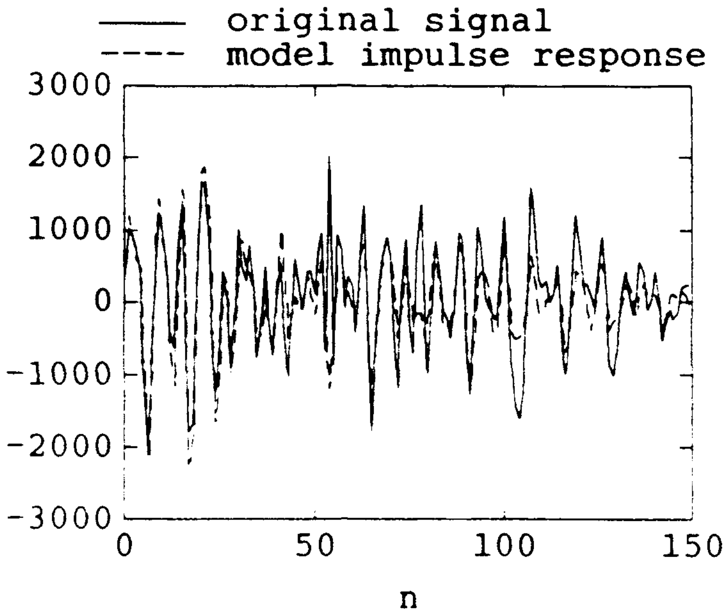


Figure 4.12: Modeling the transient Plate—model spectrums versus the original signal spectrums. (a) The iterative prefiltering model of order (12,12) and (b) the correlation domain iterative prefiltering model of order (16,16)



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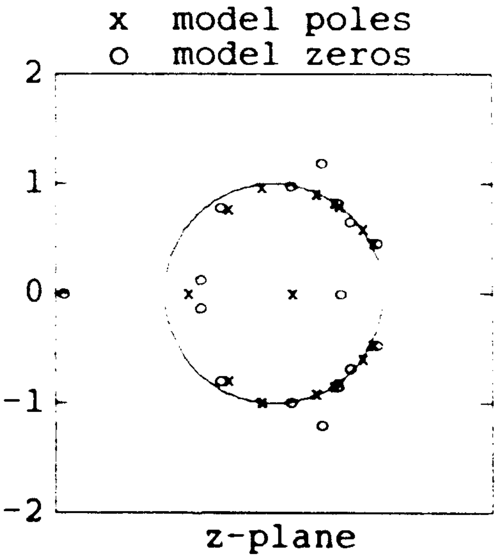
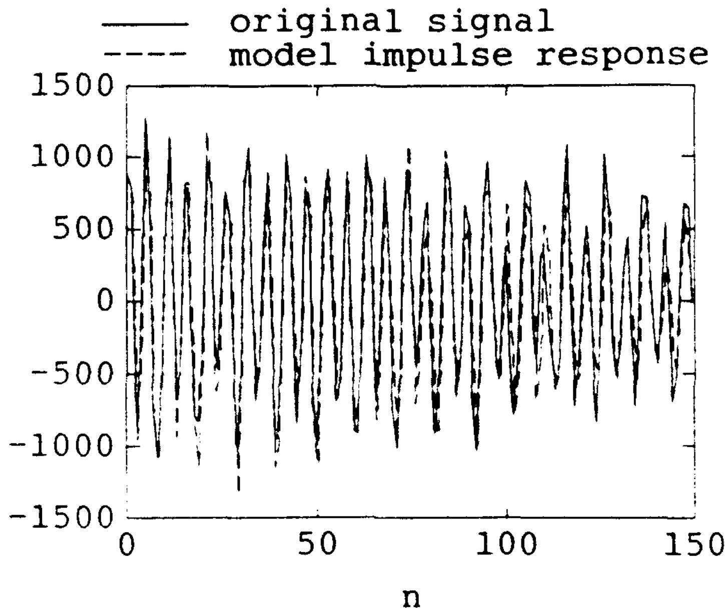


(b)

Figure 4.13: Modeling the transient Wood—Model order (16,16). (a) Correlation domain iterative prefiltering model impulse response versus the original signal and (b) the corresponding model pole-zero plot.



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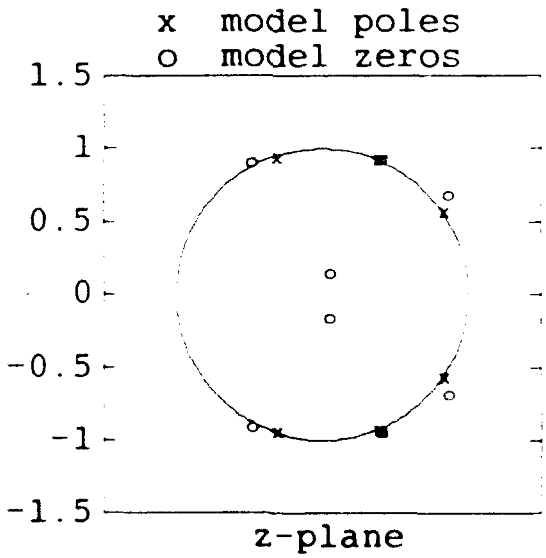

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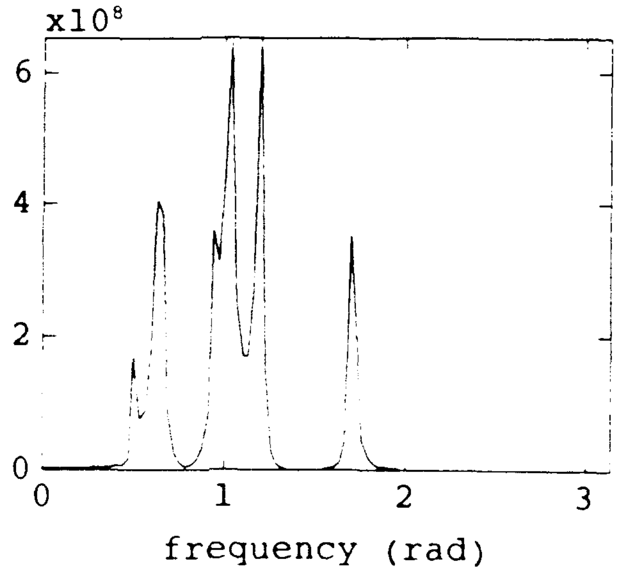
Figure 4.14: Modeling the transient Wrench—Model order (8,8). (a) Correlation domain iterative prefiltering model impulse response versus the original signal and (b) the corresponding model pole-zero plot.



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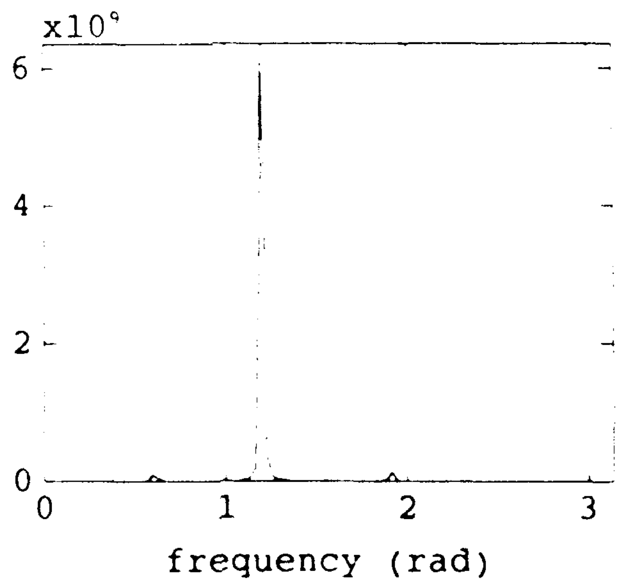
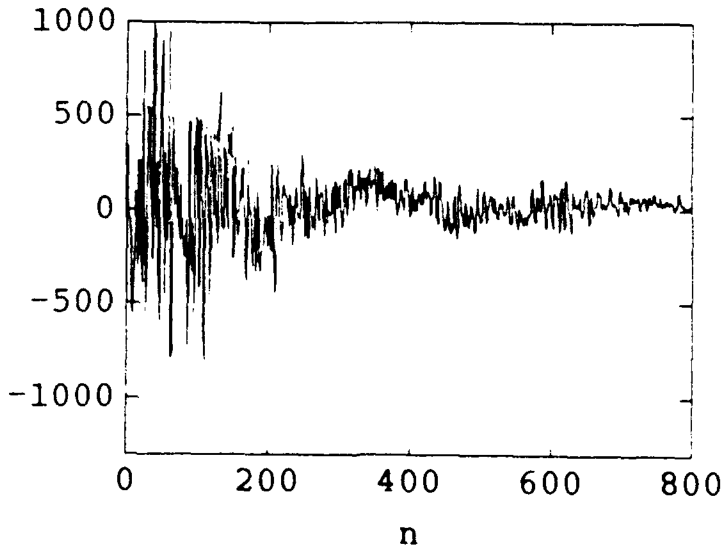


(b)

Figure 4.15: Modeling the transient Wood and Wrench—model spectrums. (a) The iterative prefiltering model of order (16,16) for the transient Wood and (b) the iterative prefiltering model of order(8,8) for the transient Wrench.



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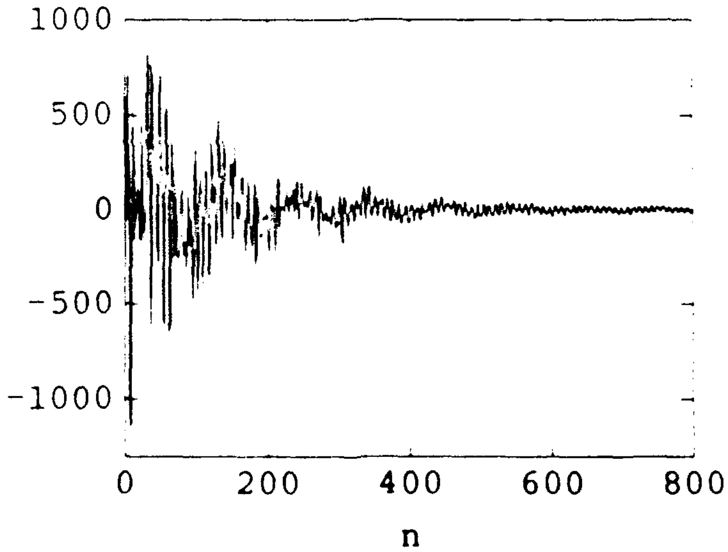

(b)

Figure 4.16: Modeling the transient Plate over the large segment (450:1250) using iterative prefiltering, model order (16,16). (a) The original Plate segment and (b) the model impulse response.

V. CONCLUSIONS

A. PERFORMANCE COMPARISON SUMMARY

The modeling of signals as the impulse response of a linear pole-zero system is an important tool in digital signal processing. One natural area for applying such an approach is in the modeling of transient, impulse response-like waveforms. The specific approach taken in this thesis was to determine which pole-zero modeling algorithms are most suited to modeling complex, real world transient waveforms. The modeling criterion emphasized is to obtain the best (least squares error) *time domain* match between the model impulse response and the original signal. This criterion was adopted because it provides a degree of signal characterization that is a step beyond normal power spectrum estimation. Indeed, the strength of pole-zero models is their ability to describe not only the resonances present (model poles), but also how these resonances are related (model zeros). Because of the widely varying characteristics anticipated for real world signals, a key evaluation criterion for any modeling algorithm is robustness. In particular, algorithms must be effective in the presence of signal degrading effects like noise and model degrading effects such as unknown model order. The performance of several selected algorithms were compared for known impulse response test sequences in Chapter III. The modeling experience gained in these experiments was then applied to modeling laboratory generated acoustic transient datain in Chapter IV as a test of 'real world' effectiveness.

Four basic algorithms were chosen for comparison: Prony's method, the least squares modified Yule-Walker equations (LSMYWE), iterative prefiltering, and the Akaike maximum likelihood estimator (MLE). An algorithm which is an extension of

iterative prefiltering into the correlation domain was also presented. For those methods in which the model poles and zeros are determined separately (Prony's method and LSMYWE), four methods for determining model zeros (i.e., transfer function numerator coefficients) were considered: the upper partition of Prony's method, spectral factorization, Durbin's method, and Shank's method. The major conclusions of the algorithm performance comparisons conducted in Chapter III and Chapter IV are as follows:

1. Algorithms that are unable to model zeros outside the unit circle (Durbin's method, Akaike MLE) have limited versatility when modeling arbitrary transient waveforms. All the acoustic transients in Chapter IV required a non-minimum phase model to obtain the best time domain match.

2. The most robust and effective method for finding zeros that gives the best least squares time domain match was found to be Shank's method. In fact, applying Shank's method as a last step improved the final model of all algorithms to some degree. The upper partition of Prony's method and spectral factorization are not very useful because of their extreme sensitivity to noisy or time shifted signals.

3. Prony's method and Akaike MLE have difficulty modeling signals in which additive noise is present. Even small amounts of additive noise causes a dramatic loss of pole frequency resolution for Prony's method. The use of excess poles and singular value decomposition were found to be effective in overcoming these effects but these methods depend on a knowledge of correct model order. Also, unlike spectrum estimation, excess poles must be retained for time domain matching. This is in direct contrast to the parsimonious use of model parameters normally provided by a pole-zero model. The difficulties encountered with the

Akaike MLE were computational in nature; for noisy signals the Akaike MLE algorithm usually terminated prematurely when either a non-minimum phase model was encountered during an iteration or when a numerical overflow was induced by an unstable model estimate.

4. LSMYWE with singular value decomposition and iterative prefiltering (including the correlation domain version) were found to be the most effective algorithms for modeling a signal when additive noise is present. If the true model order of a system is unknown, it is best to discard only one singular value. Almost all the resolution gain occurs with the first singular value. Discarding additional singular values is intended to reduce the variance of excess poles but it will cause poles to be biased if all model poles are necessary for signal modeling. Both of these methods demonstrated the consistent ability to model poles very close to the unit circle. This capability was essential when modeling the acoustic transients used in this thesis.

Combining the above observations leads to our recommended strategy for modeling an arbitrary transient signal. First, use LSMYWE with one singular value removed and Shank's method to find an initial model estimate. Next, use this estimate to initialize either iterative prefiltering or correlation domain iterative prefiltering. Finally, if desired, apply Shank's method to optimize the time domain fit of the model zeros.

B. RECOMMENDATIONS FOR FUTURE STUDY

The results of Chapter IV indicate that there are pole-zero modeling algorithms available that are sufficiently robust to be useful for modeling many complex, real world transients. A number of issues regarding the pole-zero modeling of transient signals and the application of such models require further study. These issues include:

1. The degree to which the noise performance of correlation domain iterative prefiltering may be improved by introducing noise compensation of the autocorrelation sequence requires study.
2. A study of the effectiveness of model order determination techniques when applied to transient modeling would facilitate more effective use of transient modeling techniques.
3. The effectiveness of iterative prefiltering and correlation domain prefiltering as a spectral estimation technique should be explored further.
4. The ability of pole-zero models to describe the time domain characteristics of a signal could aid in the detection of signals. Such an application needs to be persued.
5. A study of the structural relationship of pole-zero models to the specific systems that generate transients may increase the usefulness of such models.

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